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DURABILITY ANALYSIS FROM FINITE ELEMENT MODELS

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ABSTRACT

Fatigue analysis from finite element models is becoming increasingly accepted in the design process. The analysis is no longer limited to fatigue life calculations - output can now include safe working stresses, warranty claim curves, and the effects of high temperatures, manufacturing processes and assembly stresses. This paper des cribes the main features of fatigue analysis from finite element models. It shows how the subsequent fatigue analysis influences the type of FEA and the load cases modeled. The paper also discusses the choice of analysis method, the processing of fatigue loading data, and the information that can be gained from detailed analysis output.

Keywords*: fatigue, finite element analysis*

1. INTRODUCTION

Fatigue analysis from finite element models is becoming an accepted design method in industry. Recent advances in fatigue analysis, particularly in the field of multiaxial fatigue, means that fatigue analysis can be at least as accurate as other aspects of engineering simulation. Advances in software design are also reducing analysis times dramatically. This paper describes some of the guidelines which help to ensure a valid fatigue analysis result.

2. FATIGUE ANALYSIS FROM FEA

For many analyzes the principle of stress scaling is used. For a component with multiple load directions, a "reference value" of each applied load is analyzed separately in a linear elastic FEA, and the results written to a stress output file as separate stress solutions. In the fatigue software, for each node :

- The "reference load" stress tensor is multiplied by its corresponding load history, to produce
- Time histories of each stress tensor.
- The time histories of the stress tensors are superimposed.
- The time histories of the principal stresses are calculated.
- A multi-axial elastic-plastic correction is used to calculate the time history of elastic-plastic stresses and strains.
- The damage parameter (for example the time history of the shear strains on a critical plane) is calculated.

Figure 1 shows a practical application of this technique to a component with a single time history of applied loading. The component is a steel lug fitting used in an aircraft engine thrust reverser. The service load history is also shown.

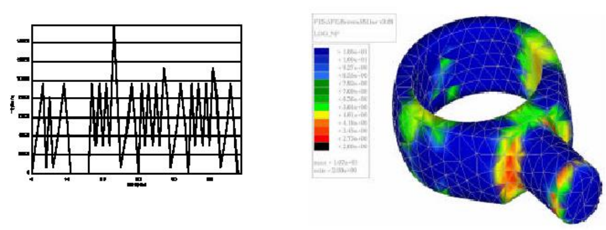


Figure 1 - Loading history for accelerated testing (left) and fatigue life contours (right). Test life 1650 flights, calculated life 1631 flights.

In an accelerated fatigue test the failure location was in the fillet radius, where the stresses are biaxial and where a pronounced stress gradient exists. The calculated fatigue life, using *fe-safe* (Safe Technology, 2004) was 1631 flights and the test life was 1650 flights. An example of multiaxial loading is the forged aluminum suspension component shown in Figure 2.

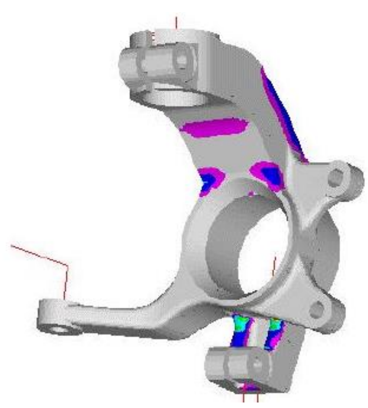


Figure 2 - Fatigue life contours for a steering knuckle.

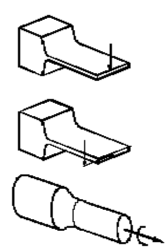
Three forces, for braking, cornering and vertical forces, are applied at the tire contact patch. Three linear elastic FE analyses are used, one for each load applied separately. These are then combined with time histories of the three loads. On an accelerated fatigue test fatigue cracks initiated in two fillet radii. The correlation between the test life (41,000 miles to significant cracking) and the calculated life to crack initiation (27,000 miles) is impressive. (Colquhoun, 2000) For engine components, it is often useful to model a sequence of events in the FEA. For example, a crank shaft may be analyzed by calculating the stresses at each 5^o of rotation of the crank shaft, through two or three complete revolutions. This need not be a linear FEA solution. The sequence of stresses may then be analyzed in the fatigue software. If a linear elastic FEA is used, other types of loading may be superimposed, perhaps using the "reference loads" methods described above. This method of analysis can also be used to analyze transient events, and may be applied to the analysis of elasticplastic FEA results. Other loading descriptions can also be used - for example loads described in the frequency domain. For a rigid component with loading described in the frequency domain, Dirlik's method (Dirlik, 1985) may be used to transform the PSD of loading into a Rainflow cycle matrix. This is the loading to be applied to a linear elastic FEA model. For flexible components modal superimposition may be used, with fatigue lives calculated from the PSD of nodal stresses (for proportional principal stresses), or from an internally - generated time-domain description of the nodal stress history. Although calculated fatigue lives are obviously important, other output results may be much more useful. If the design life of the component is specified, the software will calculate the scale factor which must be applied to the nodal stress to achieve the design life. This provides a contour plot of factors of strength, showing how much the component is over strength or under strength at each node. Probabilities of failure can also be calculated from the variability in the fatigue strength of the material combined with information on how the loading might differ from that specified. These results can show how the failure rate will increase with increased usage, providing an estimate of future warranty claims.

3. CHOICE OF FATIGUE ANALYSIS METHOD

3.1 Uniaxial fatigue

The use of uniaxial fatigue methods to analyze biaxially stressed components can give very optimistic life estimates. In "Devlukia, 1985", a welded steel bracket from a passenger car subjected to multiaxial loading developed fatigue cracks at a life much shorter than that predicted by uniaxial local strain fatigue analysis. The component had also been tested under two different service duties and uniaxial analysis failed to reproduce the relative severity of the two duties. Table 1 shows results reported from a multiaxial fatigue test program (Bannantine, 1989). The three specimens were (i) simple bending, (ii) in-phase bending and torsion and (iii) axial and torsion loading with random phase relationship. Fatigue life predictions from strain gauges using uniaxial methods were always non-conservative, with predictions up to 19 times the test life.

Table 1 - Uniaxial fatigue life predic tions for various multiaxial conditions.
(Lives are repeats of the test signal).



	TEST LIFE	PREDICTION, UNIAXIAL FATIGUE
(i)	600	5000
(ii)	200	450
(iii)	4000	53000
	1700	30000
	1000	19000

3.1.1 Principal Stress criterion

Early attempts to analyze biaxia l fatigue were based on principal stresses, using a conventional *S-N* curve. For a fatigue cycle, the stress range of $\Delta\sigma_1$, or the stress amplitude $(\Delta\sigma_1)/2$, would be used with a stress-life curve obtained by testing an axially loaded specimen. The [false] assumption in this procedure is that the fatigue life is always determined by the amplitude of the largest principal stress σ_1, and therefore that the second principal stress σ_2 has no effect on fatigue life. For a simple circular shaft loaded in pure torsion a fatigue cycle of $\pm\, \tau_{xy}$ will produce a principal stress cycle of $\pm\, \sigma_1 = \pm\, \tau_{xy}$. The use of the principal stresses therefore predicts that the fatigue strength in torsion is the same as the fatigue strength under axial loading. This is not supported by test data, as Figure 3 shows.

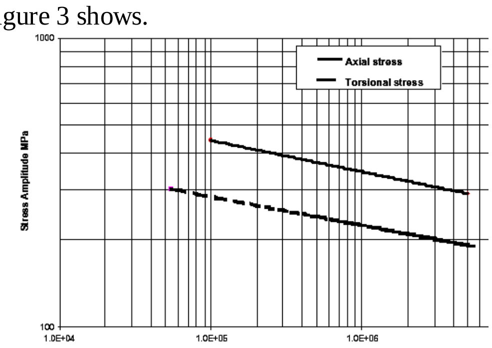


Figure 3 -Stress- life curves for axial and torsion loading

Figure 3 shows the results of fatigue tests on commonly-used steel. It is clear that the torsion fatigue strength is much lower than the axial fatigue strength - the allowable principal stress in torsion is approximately 60% of the allowable axial stress. Calculating fatigue lives using principal stress will clearly be grossly optimistic for torsion loading, and allowable torsion fatigue stresses will be overestimated by a factor of 1/0.6 = 1.66 for this material. This could mean the difference between identifying and missing a potential fatigue hot spot. (In 1927, Moore reported that, "From the quite considerable amount of test data available for fatigue tests in torsion, the general statement may be made that under cycles of reversed torsion the endurance limit for metals ranges from 40 percent to 90 percent of the endurance limit under cycles of reversed flexure". (Moore, 1927)). It has been shown over the past 20 years that principal stresses should only be used for fatigue analysis of brittle metals, for example cast irons and some very high strength steels. A fatigue analysis using principal stresses tends to give very unsafe fatigue life predictions for more ductile metals including most commonly-used steels and many aluminum alloys.

3.1.2 Principal strain criterion

This criterion proposes that fatigue cracks initiate on planes which experience the largest amplitude of principal strain. This will occur on the plane perpendicular to the surface. The standard strain-life equation for unixial stresses is

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f}{E}(2N_f)^b + \varepsilon'_f(2N_f)^c \qquad -1$$

Where,

$\Delta\varepsilon$ - *is the applied strain range*

$2N_f$ - *is the endurance in reversals*

σ'_f - *is the fatigue strength coefficient*

ε'_f - *is the fatigue ductility coefficient*

b - *is the fatigue strength exponent*

c - *is the fatigue ductility exponent.*

The maximum principal strain amplitude replaces the axial strain amplitude in this equation. Cracks are presumed to initiate on a plane perpendicular to the surface and perpendicular to the largest principal strain amplitude. In a more general case the principal stresses may change their magnitude and also change their orientation. The plane perpendicular to the surface is rotated through 180° in small steps, typically 10°. The time history of normal strain and the associated stress are calculated for each plane, and the fatigue damage is calculated for each plane. The plane with the highest calculated damage is the critical plane, and the fatigue life is the life calculated for this plane. The direction of the critical plane shows the direction of crack initiation. It is no longer presumed that cracks initiate on the plane of maximum strain amplitude but on the plane with the highest calculated fatigue damage. The SAE multiaxial test program (Tipton, 1989) used a 40mm diameter notched shaft with 5mm fillet radii, machined from SAE1045 steel. The specimens were tested under pure bending loads, pure torsion loads, and combined bending-torsion with various proportions of bending and torsion. The test results have been compared with life estimates made from measured strains at the notch. The maximum principal strain criterion produced life estimates which were non-conservative, particularly at lower values of endurance, and the scatter was large (Figure 4). Experience has shown that this criterion should be used only for fatigue analysis of brittle metals, for example cast irons and some very high strength steels.

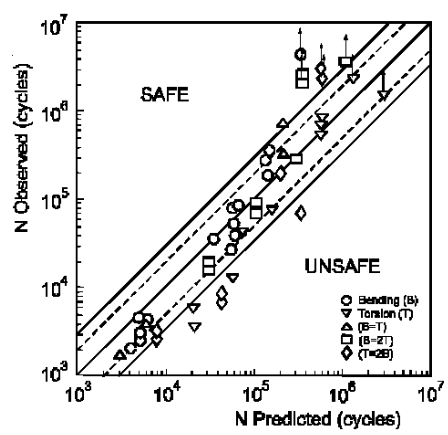


Figure 4 - SAE notched shaft test results, principal strain theory

3.1.3 Brown-Miller criterion

The Brown-Miller equation proposed fatigue cracks initiate on the plane which experiences the maximum shear strain amplitude, and that fatigue damage is a function of both this shear strain, γ_{max}, and the strain normal to this plane, ε_N

$$\frac{\Delta\gamma_{max}}{2} + \frac{\Delta\varepsilon_N}{2} = 1.65(\frac{\sigma'_f}{E})(2N_f)^b + 1.75\varepsilon'_f(2N_f)^c \quad 2$$

This formulation of the Brown-Miller parameter was developed by Kandil, Brown and Miller (Kandil, 1982). The Brown-Miller criterion is attractive because it uses standard uniaxial materials properties. Figure 5 shows the results from the SAE test program (Tipton, 1989).

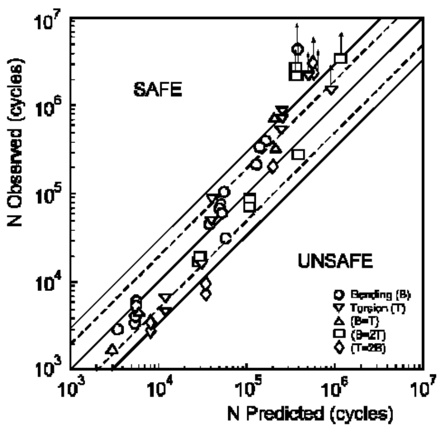


Figure 5 - SAE notched shaft, Brown-Miller parameter

In general, test results and predictions agreed to within a factor of 3. The Brown-Miller criterion is widely accepted for the analysis of most ductile metals. For plane stress conditions, if fatigue cracks initiate from the surface on planes of maximum shear strain amplitude, it has been shown that three basic planes are required (Figure 6), one perpendicular to the surface and two at 45° to the surface (Bannantine, 1989). Critical plane analysis is used when the principal stresses change their orientation during the loading history.

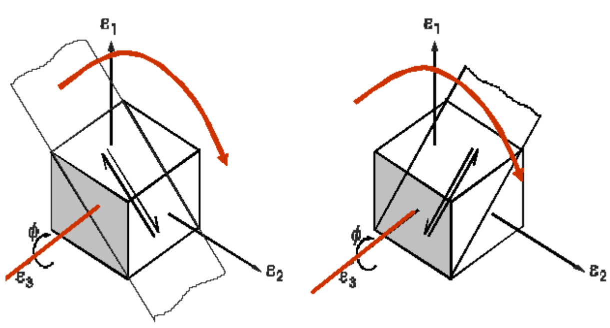


Figure 6 - The three planes used in critical plane analysis. The surface of the component is shown shaded.

Each of the three shear planes is rotated though 180° in small steps, typically 10°, and the fatigue damage calculated for each plane. The plane with the highest damage defines the fatigue life. It is no longer presumed that cracks initiate on the plane of maximum shear strain amplitude but on the plane with the highest calculated fatigue damage. Both the principal strain and Brown-Miller algorithms define the life to the initiation of a small crack. Crack initiation is becoming a common design criterion in many industries, in part because of legal liability issues. Crack growth calculations from FEA models cannot really be treated as a post-processing operation, because it is necessary to calculate stress redistribution as the crack propagates.

3.2 Effects of stress gradient and notch sensitivity

Experimental evidence from fatigue testing carried out in the middle of the last century shows that stress gradients can have an important effect on the total fatigue life of a component. However, local strain fatigue analysis from strains measured in notches has shown good correlation even though the effects of stress gradient are ignored. This suggests that stress gradients have little effect on the life to crack initiation, but a significant effect on the subsequent crack growth. This was demonstrated by Frost and Dugdale (Frost, 1957) and Frost (Frost, 1960), using test data on flat plate and round bar specimens in aluminum alloy and steel materials. They showed that if fatigue life to first crack initiation is considered, then the fatigue strength reduces with increasing stress concentration, i.e. that the crack is initiating in response to the surface strains and stresses with little or no influence from stress gradient. Shown in Figure 7.

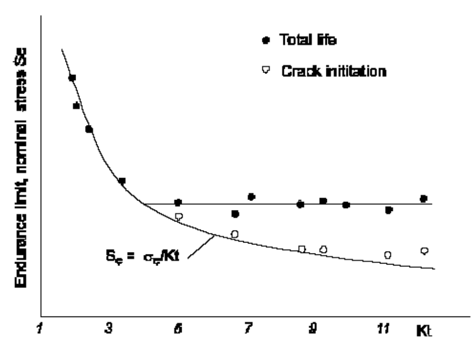


Figure 7 - Relationship between endurance limit stress and the stress concentration factor *Kt* for crack initiation and total life. (σ_e is the smooth specimen endurance limit stress)

If total life (crack initiation plus propagation) is considered, then blunt notches behave in the same way, but the life at sharp notches is significantly influenced by stress gradient effects. It seems therefore that crack initiation may be determined by surface strains and stresses with no significant stress gradient effect, at least for the geometric features usually present in engineering design. However, in the high stress gradients which are present at sharper notches, the crack may initiate but then propagate into a region of low stress where the stress intensity factor at the crack tip is less than the non-propagating value, and as a result the crack ceases to grow. Use of stress gradients may therefore imply that the designer is no longer designing to prevent crack initiation, but instead is relying on a calculation that the cracks will initiate but not propagate. This may be an unsafe assumption in some complex components where crack growth may be accelerated by load redistribution.

3.3 The Endurance Limit

Many materials exhibit endurance limit stress amplitude under constant amplitude testing. Under variable amplitude loading the endurance limit may disappear or its amplitude may be very much reduced (Conle, 1980), (DuQuesnay, 1993). Figure 8 shows a measured strain history from a truck steering arm (upper signal), and the strain history that is produced if all the cycles smaller than the constant amplitude endurance limit are removed.

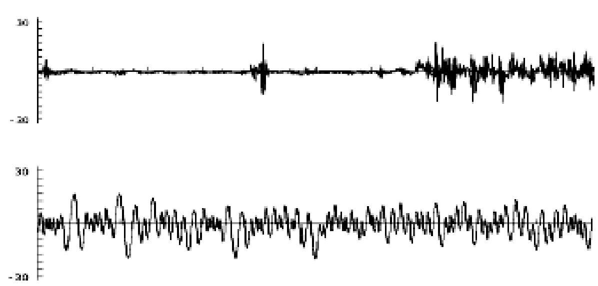


Figure 8 - Measured truck steering arm loading (top) and the same signal after omitting cycles below the endurance limit (bottom)

Fatigue testing using the truncated signal produced fatigue lives which were 9 times longer than those produced using the full signal (Kerr, 1992). It is now common practice to use an endurance limit stress or strain amplitude equal to 20% or 25% of the constant amplitude value, to allow for the damaging effect of small cycles when they are mixed in with larger cycles.

3.4 Truncating Loading Histories

It is tempting to eliminate small cycles from the loading histories before analysis, and analysis times can be reduced dramatically if this is done. The process is usually carried out by extracting the peaks and valleys from a signal, omitting those peaks and valleys that form small cycles. There are two potential pitfalls however. First, the cycle omission criterion, or "gate", must be no greater than, say, 20%-25% of the constant amplitude endurance limit. Secondly, with multichannel loading, this peak/valley process assumes that peaks and valleys in the principal stresses (or peaks and valleys in the shear strain history on the critical plane) will only be formed by peaks and valleys in the loading. In general peaks and valleys in the calculated principal stresses or the damage parameter do not correspond to a peak or valley in one of the load histories. Serious errors in the calculated fatigue lives can be produced by peak-valley extraction of multiaxial loading histories. The following example (Malton, 2004) used triaxial braking, cornering and vertical loads from the wheel of a four-wheel-drive vehicle, measured on a proving ground. Fatigue lives were calculated at a critical point on a suspension component, using a Brown-Miller critical plane multiaxial fatigue analysis. At this point on the component, each load contributed approximately equally to the total principal stresses.

With the signals scaled to give a calculated life close to the target life, the fatigue life was first calculated for the full signals, with no pre-processing of the load histories to extract peaks and valleys. Figure 9 shows the distribution of calculated damage plotted against cycle range. The calculated life was 250 laps of the proving ground circuit. The peaks and valleys were then extracted from the three signals using multi-channel peak/valley extraction. The calculated life increased slightly, to 269 laps. The damage distribution is shown in Figure 9 labeled ‘Gate 0%’.

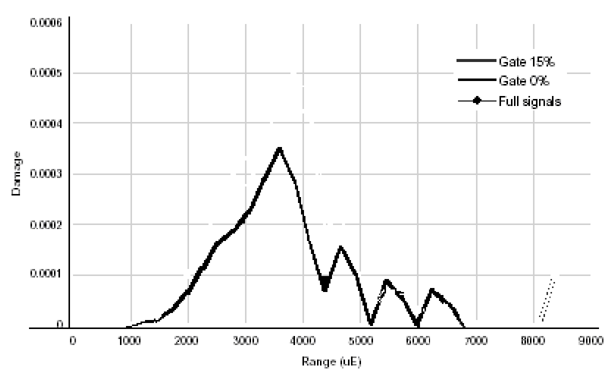


Figure 9 - Calculated fatigue damage distribution for a component with tri-axial loading, for the full signals, signals after peak-valley extraction, and with a 15% cycle range omission.

The damage for the largest cycle remains unchanged, indicating that the peak and valley for this cycle coincided with a peak and valley in the loading signals. Calculated damage at almost all other cycle ranges is different from that calculated for the full signals. The process of peak-valley extraction has therefore changed the cycle distribution across all ranges. This indicates that even when all cycles in the loading histories are retained, peak-valley extraction changes the magnitude of the cycles at the critical location on the component, and changes the calculated fatigue life. Finally, the peaks and valleys were extracted from the loading histories with a cycle omission criterion ('gate') set to exclude cycles smaller than 15% of the maximum load in each loading history. Multichannel peak-valley with cycle omission was used. The calculated life increased to 395 laps. The damage distribution is shown in Figure 9 labeled 'Gate 15%'. Again, damage is removed at all cycle ranges except the largest cycle. Calculated lives are shown in table 2. The amount by which the signals were reduced by the peakvalley procedure is also shown – the 15% gate produced loading signals which contained only 1/36th of the data points in the original signal. The analysis was repeated for the same component with the signals scaled to give a much longer life. The results, for the full signals and for the 15% cycle omission criterion, are shown in Figure 10.

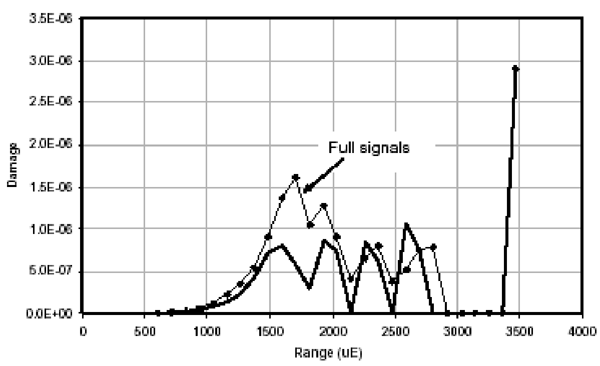


Figure 10 - Calculated fatigue damage distribution for a component with tri-axial loading, full signals (life=16700 laps) and 15% cycle range omission (life =69100 laps)

The calculated life using the full signals was 16,700 laps of the proving ground. The calculated life increased to 69,100 laps (i.e. by a factor of 4) for the peak-valley signals with a cycle omission criterion equal to 15% of the maximum load on each channel, even though Figure 10 shows that the cycle omission criterion was set below the level of the smallest damaging cycle range.

Table 2 - Effect on fatigue life of peak-valley and cycle omission applied to multi-channel loading histories.

Signal processing	Calculated life (laps of proving ground)	Data reduction factor
Full signals	250	1
Peak-valley with no cycle omission	269	1/1.5
Peak-valley with 10% gate	351	1/20
Peak-valley with 15% gate	395	1/36

3.5 Mesh Density

Fatigue from FEA is a relatively new subject, and the rules for mesh density are not fully defined. However, the fatigue results will only be as accurate as the stress information in the model, and a 5% error in stresses can result in a factor of 2 errors in the calculated fatigue life.One standard test is to compare the non-averaged nodal stresses and the stresses averaged at nodes. A difference of more than 10 percent could indicate an inadequate mesh. Figure 11 shows the effect on calculated life of a preliminary and a fully-featured mesh for the suspension component of Figure 2.

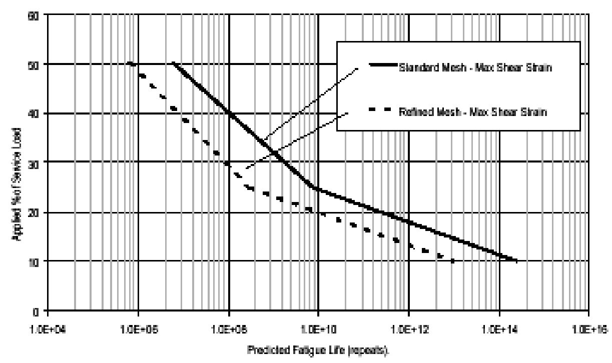


Figure 11 - Effect of mesh density on calculated fatigue life.

Figure 12. shows the Fatigue test results for butt, fillet, seem and spot welds correlated using the Battelle equivalent structural stress parameter which reflects the graph of Effect of mesh density on calculated fatigue life

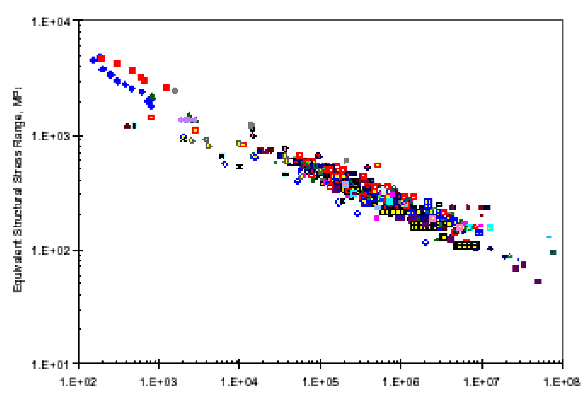


Figure 12 - Fatigue test results for butt, fillet, seem and spot welds correlated using the Battelle equivalent structural stress parameter.

Experience suggests that fatigue lives should be calculated from nodal stresses, rather than integration point stresses, because fatigue cracks usually initiate from the surface of the component. If the lives are calculated from non-averaged nodal stresses, this allows the user to assess the effect of mesh density by setting different amounts of averaging when plotting fatigue life contours. An adequate mesh density is necessary to define stress levels and degree of biaxiality in notches, and for large models sub-modeling may be necessary to achieve the required mesh quality. Compensating for inadequate mesh density by the use of additional stress concentration factors will not produce the correct biaxial stress field.

3.6 Welded Joints

The forgoing comments apply to the analysis of machined, cast and forged components. Until recently fatigue analysis of welded joints in finite element models had required much more user input and subjective judgment. This situation has been transformed by recent advances at the Battelle Memorial Institute in the United States. The Battelle mesh-insensitive structural stress method (Dong, 2001, 2002), patented as the Verity™ method, uses nodal forces to calculate structural stresses at the weld toe. The structural stress consists of three components, a membrane stress, a linear bending stress, and a self-equilibrating local notch stress. Welds contain crack-like defects, so the fatigue life is dominated by growth of these defects. The crack growth is dominated by the membrane and bending stresses, and the local notch stress can be ignored. The structural stress is modified to allow for plate thickness and type of loading (the proportion of axial and bending stresses) to produce an equivalent structural stress. It has been demonstrated by analysis of over 3000 fatigue test results that almost all weld geometries can be described by a single stress-life curve, where the stress is the equivalent structural stress. Thus a single master fatigue curve can by used for welded joints in steels, and will cover large structural welds, small welded components, seam welds, spot welds in shear and peel, etc. A separate curve applies to welds in aluminum alloys. This method offers several advantages over traditions methods.

(a) The stresses are calculated at the weld toe, not some subjective distance from it.
(b) A single S-N curve describes all welds, so welded joints need not be 'classified'.
(c) The method has been shown to be very mesh-insensitive.

The Verity™ method is available as an additional module in *fe-safe*. This allows a single contour plot to show the fatigue lives for the welds and non-welded areas of a component.

4. CONCLUSIONS

This paper has discussed some of the guidelines to be followed when performing durability analysis from finite element models. Research in the past 15 years, particularly in the field of multiaxial fatigue, has transformed the accuracy of fatigue analysis. With care the results can be at least as accurate as other aspects of engineering simulation. Analysis times are also reducing. For example, a model comprising 700,000, 4-node elements in a 3 gigabyte ANSYS file, cycled between two load steps representing cold and hot conditions, completed in 35 minutes in *fe-safe*. In the same software, a series of 36 load steps contained in an 8 gigabyte ANSYS results file completed the read-in, analysis and export in less than 90 minutes on a Windows PC. The analysis was a Brown-Miller critical plane multiaxial fatigue analysis.

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