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DYNAMIC ANALYSIS OF PISTON, CONNECTING ROD, AND CRANK OF AN I.C.ENGINE USING A MATHEMATICAL APPROACH

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ABSTRACT

The dynamic motion analysis is a complex and tedious work, which requires a complete model. The paper presents a mathematical model developed from the basics by considering the forces acting on the crank, connecting rod and piston, for both primary and secondary motions. The developed model was used to study the variation of the forces at crank pin joint, which is the base for designing a crank. The secondary motion of the piston is modeled by considering the different forces acting on the piston. In order to impose the effect of hydrodynamic lubrication the 1D Reynolds equation is solved for fully flooded lubrication condition. The variation of forces should be known before the design of any component, to know this analytical approach is a complex, time consuming but inexpensive tool. The model developed will help to study the dynamic performance of the engine before the manufacture.

Keywords: *Dynamic analysis, Piston Secondary Motion, Piston tilt, Lateral piston movement*

1. Introduction

The process of combustion and dynamic motion of piston assembly component plays an important role in developing an environment friendly I.C.Engine. The dynamics of piston assembly and liner system has a considerable effect on the lubrication conditions between the piston skirt and cylinder liner.

One of the effects of dynamic phenomena is to affect the relative angle between the ring and the liner, which alters the position of minimum point on the ring-running surface. If the position of the minimum point changes the amount of space available for oil to fill between the ring and the liner will change and therefore the friction and lubrication condition will change. Here it makes it necessary to have a model, which describes the dynamics of piston assembly in detail.

The work of modeling piston motion started somewhere in 1960's when Ungar and Ross [1] published their work on the subject. These early models considered only dynamics of the system, neglecting the hydrodynamic interaction between skirt and liner. Li et.al [2] made another attempt by modeling the system by combining piston and piston pin in to a lumped mass. The hydrodynamics of skirt-liner interface considered and 1D Reynolds equation was solved for the thrust and anti thrust-side of the piston. The model was used to study the effects of pin offset, clearance and viscosity on secondary motion.

The analysis of engine friction distribution of the engine shows that the piston assembly has a high share at the total engine friction. This offers a high potential to optimize piston assembly friction [3]. Knoll and Peeken [4] considered this work with hydrodynamics of skirt –liner interaction; they formulated and solved 2D Reynolds equation for oil film using FEM. They assumed that skirt was fully loaded with oil and did not consider surface roughness or boundary lubrication. The measurement of piston and piston ring assembly was carried out by Takaharu et al [5].

The first attempt in modeling the dynamics of articulated piston was presented in 1992 by Dursukaya and Keribar [6, 7]. In this, each component of articulated piston was treated as a separate component by developing equation of motion for each. The resulting 10 d.o.f systems were solved in conjunction with skirt elastohydrodynamic equations, but the effects of skirt surface roughness and waviness parameters were not considered. After this work number of models were developed to describe the piston and piston ring dynamics and to study the effects [8, 9, 10, 11]. The analysis of skirt hydrodynamic and boundary lubrication i.e. calculation of oil and asperity contact pressure, was either ignored or simplified in early piston motion

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models. It is now increasingly recognized that transient skirt elastic deformations during the engine cycle are another key factor in piston skirt lubrication.

Jang and Cho [12] performed a study in 2004 to know about the piston movements in bore clearance by changing the skirt profile and piston offset. A mixed lubrication model based on a two-dimensional Reynolds equation is developed and presented to use in conjunction with a piston secondary motion analysis.

Kurbet et. al [13] developed three-dimensional finite element models of single-cylinder and four-cylinder internal combustion engines to analyze the piston and ring motions. The ring motion in the radial and axial directions and the ring twist, along with the end gap variation, are studied in detail. They concluded that piston tilt has a profound effect on the end gap variation, the ring twist and the ring lift.

In another study conducted by Kurbet et.al [14] developed three-dimensional finite element models of single-cylinder and four-cylinder internal combustion engines to estimate the blowby.

Kurbet and. Malagi [15] made detailed review study on the piston and ring dynamics and concluded that in order to understand the complete complex dynamics of the system, a three-dimensional model is required.

2. Model Description

The piston motions in the cylinder bore of an Internal Combustion Engine are separated in to two distinct components as primary and secondary motions. The paper describes the equation of motions for both.

2.1 Primary motion

The primary motion refers to the reciprocating motion of the piston and this motion is uniquely determined by the design data of the engine like, the stroke, connecting rod length and engine speed etc.

When crank is turned through an angle θ, the displacement (Y) can be obtained from the geometry shown in Fig.1 as

Taking (l/r) = n, Y can be expressed as

$$Y = r[(1-\cos\theta) + (n + \sqrt{n^2} - \sin\theta)] \qquad (1)$$

The angular velocity and angular acceleration of the crank may be obtained by

$$\omega = \frac{d\theta}{dt} \quad \text{and} \quad \alpha = \frac{d\omega}{dt} \qquad (2)$$

The velocity ($\dot{Y}$) and acceleration ($\ddot{Y}$) of the piston at any instant can be found by differentiating the above two equations, and these are expressed by the equations (3) and (4).

$$\dot{Y} == \frac{(r\sin\theta + C_p)(r\omega\cos\theta) + S(r\omega\sin\theta)}{S} \qquad (3)$$

$$\ddot{Y} = \frac{(r\cos\theta.\omega^2)}{S} - \frac{(r\sin\theta. + C_p.)(r\sin\theta.\omega^2)}{S} + \frac{(r\sin\theta + C_p)^2(r\cos\theta.\omega)^2}{S^3} + r\cos\theta.\omega^2 \qquad (4)$$

The angular velocity and acceleration of the connecting rod can be expressed as equation (5),

$$\omega_r = \frac{(\omega * r * \sin\theta)}{\sqrt{(l^2 - r^2 * \sin\theta^2)}} \qquad (5)$$

$$\alpha_2 = -\omega^2 * \sin\theta\left[\frac{n^2 - 1}{(n^2 - \sin\theta^2)^{3/2}}\right] \qquad (6)$$

The linear velocity of the center of mass of the connecting rod in both in X and Y direction are expressed by equation (7a) and (7b)

$$V_{gy} = -r\omega\sin\theta - \frac{L_g\omega\sin 2\theta}{2 * n\sqrt{n^2 - \sin\theta^2}} \qquad (7a)$$

$$V_{gx} = r * \omega * \cos\theta - \frac{L_g * r * \omega\cos\theta}{l} \qquad (7b)$$

The differentiation of equation (7a & 7b) with respect to time will result in the linear acceleration of the center of mass of the connecting rod in both X and Y direction.

$$a_{ry} = -r\alpha\sin\theta - r\omega^2\cos\theta - \frac{\alpha L_g\sin 2\theta}{2n\sqrt{(n^2 - \sin\theta^2)}} + \frac{\omega L_g\sin 2\theta^2}{4n * (n^2 - \sin\theta^2) * \left(\sqrt{n^2 - \sin\theta}\right)} \qquad (8)$$

$$a_{rx} = r\alpha\cos\theta - r\omega^2\sin\theta - \left[\frac{\alpha L_g\cos\theta + L_g\omega\sin\theta}{n}\right] \qquad (9)$$

The forces acting X direction will be given by

$$F_{py} = m_p * \ddot{Y} + \pi * R_p{}^2 * P_g \qquad (10)$$

The forces on the joint between the connecting rod and crank are obtained by solving the equation of forces in X and Y direction and moment about the center of the mass of connecting rod. These forces are given by equation (11, 12).

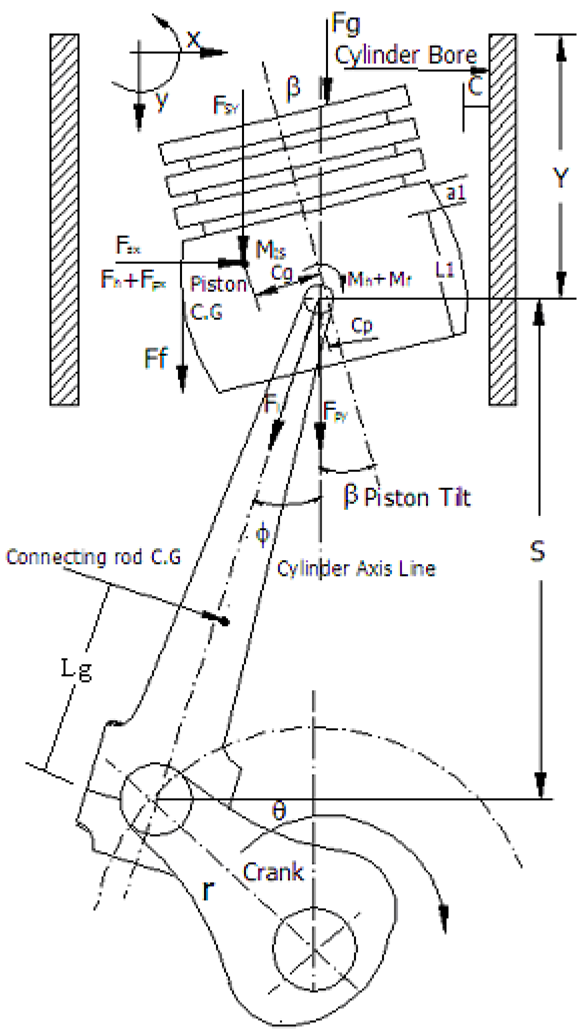


Fig. 1 Forces and Moment Acting on the Piston Assembly

$$F_{ay} = m_r a_{ry} + F_{py} \qquad (11)$$

$$F_{ax} = \frac{1}{l}\left[\frac{I_{zz}\alpha_2 - F_{ax}L_g - F_{px}\left(l - L_g\right)}{\cos\phi} + m_r * a_{ry}\left(l - L_g\right)\right] \qquad (12)$$

The forces F_{ax} and F_{ay} are expressed in a coordinate system attached to the crankshaft as equation (13 & 14).

$$F_X = F_{ax}\cos\theta + F_{ay}\sin\theta \qquad (13)$$

$$F_Y = F_{ay}\cos\theta - F_{ax}\sin\theta \qquad (14)$$

2.2 Secondary motion of the piston

Piston Secondary motion consists of a translation motion perpendicular to the cylinder axis and a rotation about the wrist pin axis. During the operating cycle of the engine certain moments and lateral forces are generated, which act on the piston (as shown in Fig. 1 and Fig. 2) and causes the piston secondary motion due to the presence of small clearance between the piston and cylinder [1,2,7,8,15,16].

In the formulation of the equation of motion, the axial and lateral force balance is written in the traditional way.

Forces acting in Y- direction

$$\sum F_y = F_g + F_f + F_{py} + F_{sy} + F_l\cos\phi = 0 \qquad (15)$$

Forces acting in X- direction

$$\sum Fx = F_h + F_{px} + F_{sx} - F_l\sin\phi = 0 \qquad (16)$$

$$\sum M_{pin} = 0$$

i.e. $M_h + M_f + M_{IS} + F_{sx}(a-b) - F_{sy}.C_g + F_g.C_p = 0$ (17)

From Equation (6)

$$Fl = \frac{F_h + F_{px} + F_{sx}}{\sin\phi} \qquad (18)$$

Using F_l in equation (15) and considering

$$F_w = (F_g + F_{py} + F_{sy})\tan\phi \quad \& \quad M_w = F_g C_p - F_{sy} C_g$$

Then, we can write

$$F_w + F_h + F_f\tan\phi = -F_{px} - F_{sx} \qquad (19)$$

$$M_h + M_f + M_w = -M_{IS} - F_{sx}(a-b) \qquad (20)$$

The Axial Forces on the piston due to the piston movement is given by

$$F_{sy} = -m_{pis}\ddot{Y} \tag{21}$$

$$F_{py} = -m_{pin.}\ddot{Y} \tag{22}$$

The Translational acceleration at top $(\ddot{e}_t)$ and bottom $(\ddot{e}_b)$ portion of the piston is obtained by the geometrical relation.

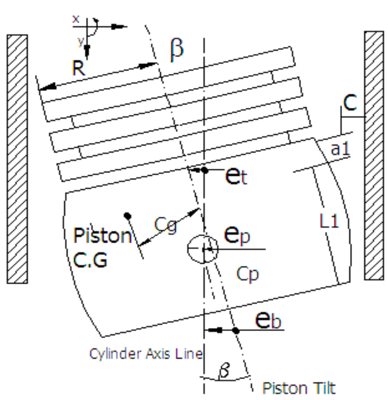


Fig. 2 Lateral Displacement of Piston at Top and Bottom

$$\ddot{e}_p = \ddot{e}_t + \frac{a(\ddot{e}_b - \ddot{e}_t)}{L} \tag{23}$$

The acceleration of piston skirt mass

$$\ddot{e}_{c.g} = \ddot{e}_t + \frac{b*(\ddot{e}_b - \ddot{e}_t)}{L} \tag{24}$$

The inertia forces in x-direction
Due to translational movement of piston

$$F_{sx} = -m_{pis}(\ddot{e}_t - \frac{a}{L}(\ddot{e}_b - \ddot{e}_t)) \tag{25}$$

Due to rotational movement of piston

$$F_{px} = -m_{pin}(\ddot{e}_t - \frac{a}{L}(\ddot{e}_b - \ddot{e}_t)) \tag{26}$$

Moment due to skirt inertia force

$$M_{IS} = I_{pis}\frac{(\ddot{e}_t - \ddot{e}_b)}{L} \tag{27}$$

The final equation of motion for the piston can be expressed the below matrix form

$$\begin{bmatrix} m_{pin}\left(1-\frac{a}{L}\right)+m_{pin}\left(1-\frac{b}{L}\right) & m_{pis}\left(\frac{a}{L}\right)+m_{pin}\left(\frac{b}{L}\right) \\ \frac{I_{pis}}{L}+m_{pin}(a-b)\left(1-\frac{a}{L}\right) & \frac{-I_{pis}}{L}+m_{pin}(a-b)\left(\frac{b}{L}\right) \end{bmatrix}$$

$$\begin{bmatrix} \ddot{e}_t \\ \ddot{e}_b \end{bmatrix} = \begin{bmatrix} F_h + F_f \tan\phi + (F_g + F_{py} + F_{sy})\tan\phi \\ M_h + M_f + F_g C_p - F_{sy} C_g \end{bmatrix} \tag{28}$$

The above equation can be used to study the locus of piston motion. The forces F_h, F_f and moments M_h, M_f are obtained from the hydrodynamic action of the oil film between the skirt and liner.

3. Lubrication and Friction Analysis

In order to know the condition of piston skirt supported by oil film, the Reynolds equation is solved by using finite difference method. The two dimensional Reynolds equation for incompressible lubricants is given by equation (30)

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial y}\right) = 6U\frac{\partial h}{\partial t} + 12\frac{\partial h}{\partial t} \tag{29}$$

In this, μ and U are constants. The array of oil film thickness value ‘h’ is initially treated as a known independent variable, and the system of equation is solved for the map of dependent pressure variable ‘P’.

The oil film thickness between the skirt and liner can be approximated as

$$h = C + e_t \cos\phi + (e_b - e_t)\frac{y}{L}\cos\phi + k(x) \tag{30}$$

$k(x)$, is a function of the skirt profile measured from cylinder reference. The forces and moments due to hydrodynamic action can be obtained by obtaining the hydrodynamic pressure by solving the Reynolds -equation by assuming the full hydrodynamic lubrication condition at piston skirt. The different forces and moments can be expressed as

$$F = \iint p\cos\phi d_x d_y \tag{31}$$

$$F_f = \iint \tau d_x d_y \tag{32}$$

$$M = \iint p(a-y)\cos\phi\, d_x\, d_y \qquad (33)$$

$$M_f = \iint \tau * R\cos\phi\, d_x d_y \qquad (34)$$

3.1 Numerical methodology

The numerical solution of equation (28) starts with prescribed values of $\dot{e}_t = \dot{e}_b = e_t = e_b = 0$, because the trajectory inside the cylinder is periodic, the converged solution does not depend on the initial guess. An implicit formulation is employed here and from the initial values the crankshaft angle is advanced and from the geometrical parameters and the equations for the kinematics of the crankshaft, connecting rod, and piston velocity and acceleration is determined for $(\theta + \delta\theta)$ in time domain. An iterative process is then needed to determine the piston radial position and velocity at $(\theta + \delta\theta)$.This is performed by using Newton iterative algorithm for every 10 degree crank angle.

4. Results and Discussions

The results to be presented were obtained for a typical diesel engine. Some input parameter of the diesel engine is shown in Table 1. The results of MATLAB programming were used to plot linear and angular velocity and acceleration of piston assembly components. The Fig. 4 and Fig. 5 show linear velocity and acceleration of piston and angular acceleration of connecting rod.

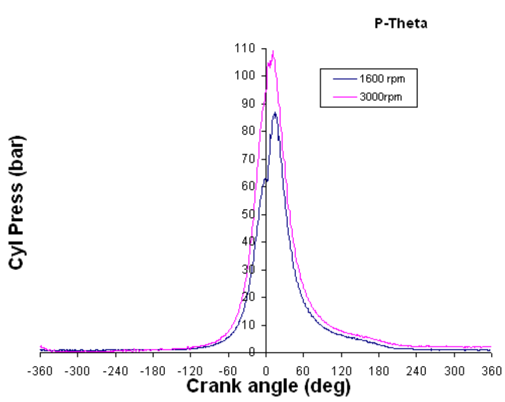


Fig. 3 Pressure Crank Angle Diagram

The Fig. 6 shows the linear acceleration of connecting rod. This plots show that, the variation of linear velocity and acceleration of piston and connecting rod from 0 degree to 360 degree are identical to their variation from 360 degree to 720 degree.

Table1: Input Parameters

a	28 mm	m_r	2.37Kg
a1	21.6 mm	Cg	0 mm
b	0.089 mm	Ipis	0.028 N-m2
c	20*e-3 mm	Lg	35.35 mm
l	120 mm	N	1600 rpm
r	55 mm	μ	0.025 Pa s
mp	0.984 Kg	Izz	0.014N-m2
mpin	0.5 Kg		

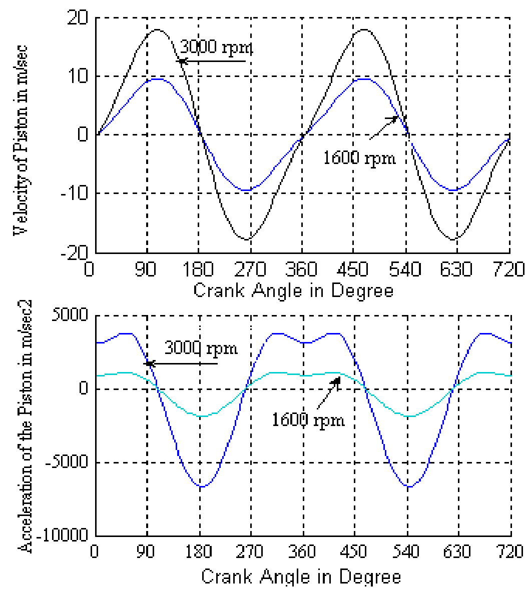


Fig. 4 Linear Velocity and Acceleration of Piston

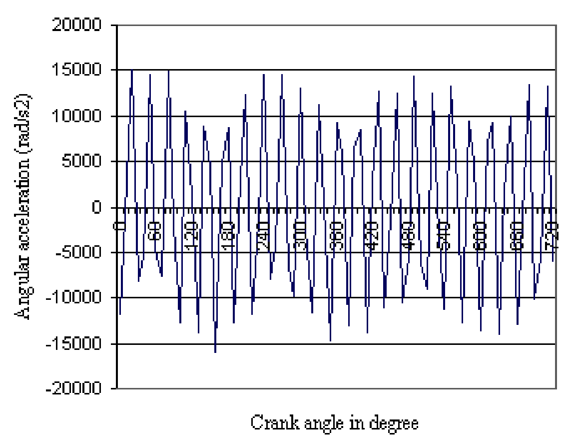


Fig. 5 Angular Acceleration of Connecting Rod

The Fig. 7(a) and Fig.7(b) shows the variation of the forces at the crank end of connecting rod defined in

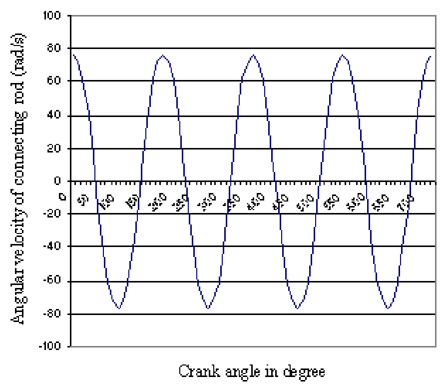
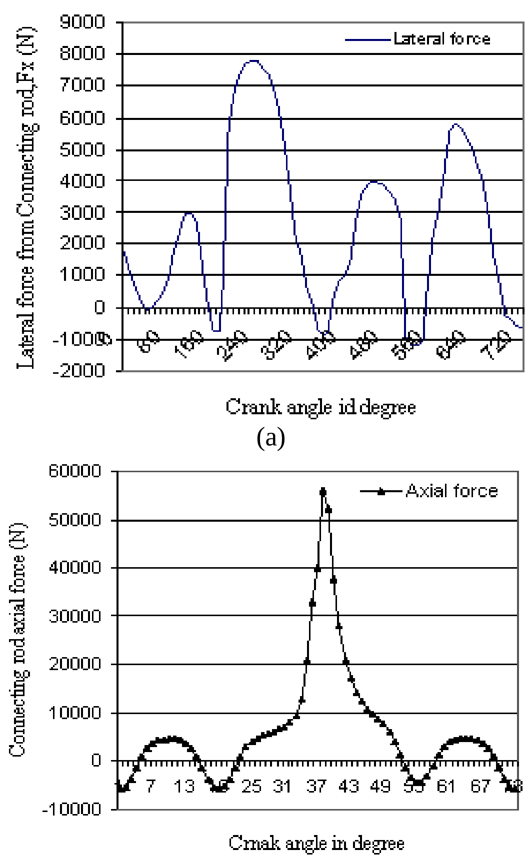


Fig. 6 Angular Velocity of Connecting Rod

global rotating coordinate system. The force F_x is that force which causes the bending of the connecting rod during the operation. F_y**,** is that force which causes the torsion on the crank shaft. The plot shows that maximum loading happens at the crank angle of 365 degree, where the combustion takes place.



(b)

Fig 7 Variation of Force at Crank End of Connecting Rod in Lateral and Axial Direction

The Fig. 8 shows the calculated piston lateral motion for one engine cycle, with different minimum rigid clearances. We can see that at zero crank angle, when the piston is at top dead center, the piston will move from thrust side (positive lateral position) to anti-thrust side (negative lateral position). This is because now the engine is at intake stroke, and the connecting rod pulls the piston down, and due to the negative connecting rod angle, the piston is pulled to anti-thrust side. Then at about mid stroke the inertia force will change the sign (the piston will change from accelerating to decelerating) and the piston then moves to the about 280 degree crank angle. It is because it is power stroke during which, load on the top of the piston is very large and this force-accompanied with the connecting rod angle will push the piston to the thrust side. In this position, since the load and correspondingly the side force is very large, the piston skirt will have very large deformation and hence result in large lateral movement.

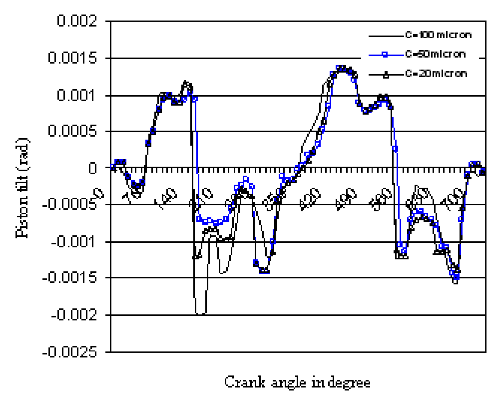


Fig. 8 Effect of Radial Clearance on Piston Lateral Movement

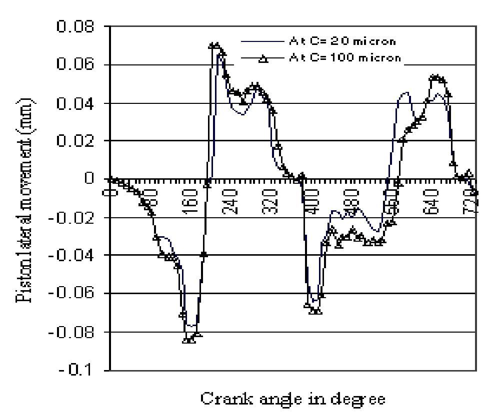


Fig. 9 Effect of Radial Clearance on Piston Tilt

The Fig. 8 and Fig. 9, shows the effect of rigid radial clearance on the piston motion. It is clear from the plots that, as radial clearance increases the piston tilt increases. Also it can be observed that piston lateral motion and piston tilt increase with rigid minimum clearance, which is understandable. Since the rigid minimum clearance get larger, the piston will move more laterally in order to have the same magnitude of deformation (which in some sense represents the side load).Generally, the piston lateral motion is driven by the side force from the wrist pin.

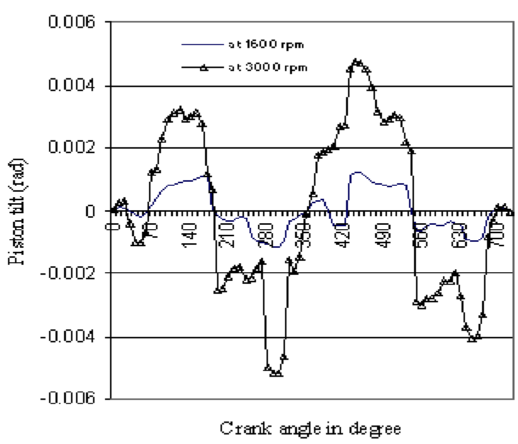


Fig. 10 Effect of Speed on Piston Tilt

The Fig. 10 shows the effect of engine speed on the piston tilting motion. It is clear as speed increases the piston tilt increases.

Fig. 11, Fig. 12 show the piston lateral force and piston motion with different engine speeds Here, we can see that with the larger engine speed the maximum piston lateral motion (which happens at about 280 degree crank angle) gets smaller. At higher speeds more viscous force will act on the piston skirt.

This is because as the engine speed increases, the sliding speed of the piston skirt over the cylinder liner increases. It is clear from Reynolds equation that a larger sliding speed will help to develop larger hydrodynamic pressure with other parameters the same, so the piston only needs to move less to generate the same amount of hydro-pressure to balance the side force from the wrist pin.

The Fig. 13 and Fig. 14 show the effect of piston pin offset on the piston secondary movement. When Cp=0 mm piston tilt is smaller than that of Cp=+0.5 mm and Cp=-0.5 mm.

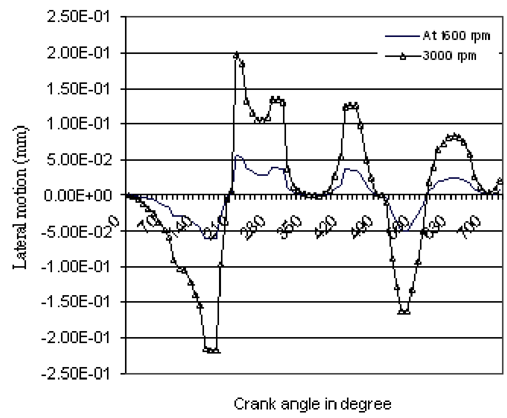


Fig. 11 Effect of Speed on Piston Lateral Movement

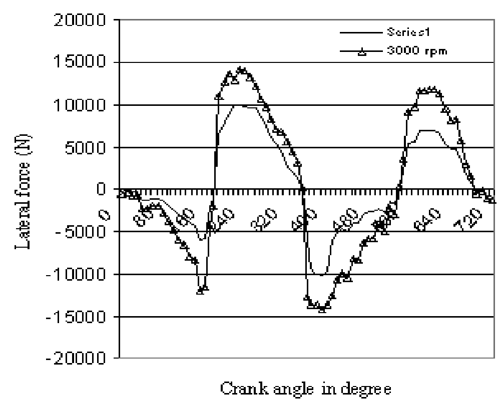


Fig. 12 Effect of Speed on Lateral Force Acting on Piston

The variation of viscous friction and hydrodynamic fluid force acting on piston skirt are shown in Fig. 15 and Fig. 16 respectively. It is seen that as speed increases the hydrodynamic frictional force will also increases.

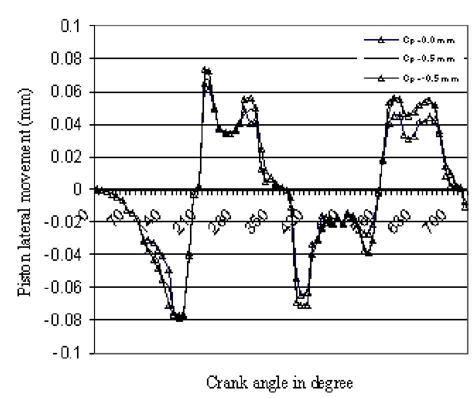


Fig. 13 Effect of Piston Pin Offset on Lateral Movement

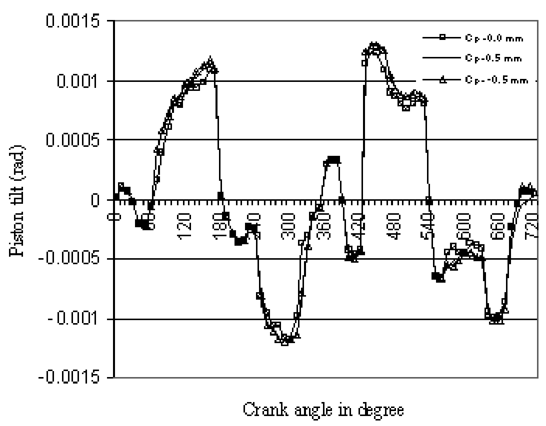


Fig. 14 Effect of Piston Pin Offset on Piston Tilt

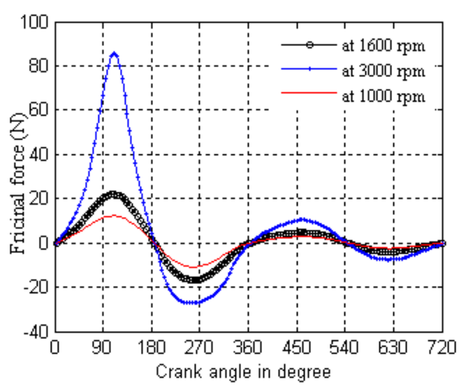


Fig. 15 Variation of Hydrodynamic Frictional Force

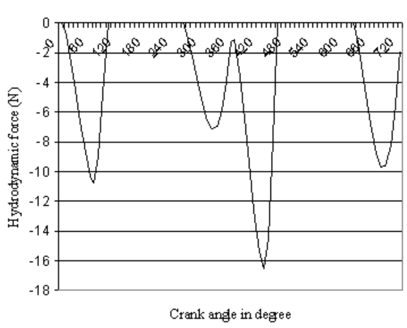


Fig. 16 Variation Hydrodynamic Force Acting on Piston at 1600 rpm

5. Conclusion

Based on the computational study, the following conclusions are arrived at the study.

i) The force acting at the crank pin joint of connecting rod will be more at around 365 degrees of crank angle.

ii) Results show that the engine speed, piston pin offset, and radial clearance all play an important role in determining the secondary motion of the piston assembly. For high engine speed and small radial clearance secondary motion tends to decrease at around 280 degree crank angle.

iii) In this study, the fully flooded hydrodynamic conditions are considered for the skirt lubrication. But the model can be made still more realistic by considering boundary lubrication.

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Nomenclature

Symbol	Meaning	Unit
a	Vertical distance from the top of piston skirt to the piston center of mass	mm
b	Vertical distance from the top of piston skirt to the to the wrist pin	mm
et	Eccentricity of piston at top of skirt	mm
e_b	Eccentricity of piston at bottom of skirt	mm
p	Fluid pressure	N/m^2
r	Crank radius	mm
t	Time	sec
C	Radial clearance	mm
C_p	Wrist pin offset	mm
C_g	Distance of piston center mass from the wrist pin position	mm
F_f	Viscous friction force	N
F_h	Hydrodynamic fluid force	N
F_l	Force acting along the connecting rod	N
F_g	Gas force generated on piston crown	N
F_{sx}	Inertia (lateral) force due to piston skirt -mass in x direction	N
F_{px}	Inertia force due to wrist pin mass (x--direction	N
F_{sy}	Axial inertia force due to skirt mass (y--direction)	N
F_{py}	Axial inertia force due to wrist pin mass (y-direction)	N
FX, FY	Force at crank end of connecting rod	N
I_{PIS}	Moment of inertia of the piston about the center of mass	$N\text{-}m^2$
K(x)	Piston skirt profile function	
L	Piston skirt length	mm
L_g	Distance of center of mass of connecting rod	mm
M_h	Moment about the wrist pin axis due to -the hydrodynamic force	N-m
M_f	Moment due to viscous friction force	N-m
M_{IS}	Moment due to skirt inertia force	N-m
M_{IP}	Moment due to wrist pin inertia force from crank end	N-m
R	Piston radius	m
V_{gx}, V_{gy}	Linear velocity of center of mass of connecting rod in x and y direction	m/s
a_{rx}, a_{ry}	Linear acceleration of center of mass of connecting rod in x and y direction	m/s^2
μ	Viscosity of oil	Pa s
τ	Shear stress	N/m^2
β	Piston tilt angle	deg
$\ddot{Y}$	Linear acceleration of piston	N/m^2
Φ	Angle between the connecting rod and piston moving direction	deg
α_1, α_2	Angular acceleration of crank, connecting rod.	m/s^2