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DESIGNING OF VARIABLES REPETITIVE GROUP SAMPLING PLAN INDEXED BY POINT OF CONTROL

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ABSTRACT

The present paper deals with the procedures for designing of variables repetitive group sampling plan indexed by indifference quality level and the relative slope on the operating characteristic curve. A table is also developed for the selection of parameters of the plan.

Keywords: *Acceptance Sampling, Indifference Quality Level, OC Curve, Relative Slope.*

1. Introduction

Acceptance sampling is a statistical tool used to make decisions concerning whether or not a lot or batch of products should be released for consumer's use. Variables sampling plans constitute one of the major areas of the theory and practice of acceptance sampling. The primary advantage of the variables sampling plan is that the same operating characteristic (OC) curve can be obtained with a smaller sample size than would be required by an attributes sampling plan. When destructive testing is employed, variables sampling is particularly useful in reducing the costs of inspection.

Another advantage of variables sampling plan is that measurements data usually provide more information about the manufacturing process or lot than do the attributes data. Generally, numerical measurements of quality characteristics are more useful than simple classification of the item as conforming or non-conforming. It is also to be pointed out that when acceptable quality levels (AQLs) are very small, the sample size required by the attributes sampling plans is very large. Under these circumstances, there may be significant advantages in switching to variables sampling plans. Thus, as many manufacturers begin to emphasize allowable numbers of non-conforming parts per million (ppm), variables sampling becomes very attractive.

Repetitive group sampling (RGS) plan is one of the attributes sampling plans developed by Sherman [1]. The operation of this plan is similar to that of the sequential sampling plan. Sherman has pointed out that the RGS plan will give an intermediate in sample size efficiency between the single sampling plan and the sequential sampling plan.

Balamurali and Jun [2] extended the concept of variables sampling to Repetitive group sampling and the resulting plan is designated as Variables Repetitive Group Sampling (VRGS) plan. The advantages of the variables RGS plan over variables single sampling plan, variables double sampling plan and attributes RGS plan have been discussed by Balamurali and Jun [2]. They have also developed tables for the selection of parameters of known and unknown standard deviation variables repetitive group sampling plan indexed by acceptable quality level and limiting quality level.

In this paper, an attempt is made to develop procedures for the selection of VRGS plan indexed by indifference quality level (IQL) or point of control and its relative slope on the operating characteristic (OC) curve. A table is also developed for the easy selection of parameters of the VRGS plan for specified p_0 (IQL) and its relative slope h_0 on the OC curve.

2. Variables Repetitive Group Sampling Plan

The VRGS plan can be applied to the measurable characteristics under the conditions of normal probability model. The following assumptions should be valid for the application of the variables RGS plan.

i. Lots are submitted for inspection serially, in the order of production from a process that turns out a constant proportion of non-conforming items.
ii. The consumer has confidence in the supplier and there should be no reason to believe that a particular lot is poorer than the preceding lots.

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In addition to it, the usual conditions for the application of single sampling variables plans, with known or unknown standard deviation, should also be valid.

2.1 Operating procedure

Suppose that the quality characteristic of interest has the upper specification limit (U) and follows a normal distribution with mean μ and known standard deviation σ. Then the following operating procedure of the VRGS plan is employed.

Step 1: Take a random sample of size n_σ, say $\left(x_1, x_2 ... x_{n_\sigma}\right)$ and compute

$$v = \frac{\left(U - \overline{X}\right)}{\sigma}, \text{ where } \overline{X} = \frac{1}{n_\sigma}\sum_{i=1}^{n_\sigma} x_i .$$

Step 2: Accept the lot if $v \geq k_{a\sigma}$ and reject the lot if $v < k_{r\sigma}$. If $k_{r\sigma} \leq v < k_{a\sigma}$, then repeat the steps 1 and 2. (Note: $k_{a\sigma} > k_{r\sigma}$)

Thus, the variables RGS plan has the parameters namely the sample size n_σ, and the acceptable criterion $k_{a\sigma}$ and rejection criterion $k_{r\sigma}$. If $k_{a\sigma} = k_{r\sigma}$ then the proposed plan is reduced to the variables single sampling plan.

3. Operating Characteristic Function

3.1. Known sigma plan

The fraction nonconforming in a lot will be defined as:

$$p = P\{X > U/\mu\} = 1 - \Phi\left(\frac{U-\mu}{\sigma}\right)$$

$$= 1 - \Phi(y) = \Phi(-v) \quad (1)$$

Where $\Phi(y)$ is given by

$$\Phi(y) = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-z^2}{2}\right) dz \quad (2)$$

Let P_a and P_r respectively be the probability of accepting and rejecting a lot based on a single sample under VRGS plan with parameters $(n_\sigma, k_{r\sigma}, k_{a\sigma})$ when the fraction nonconforming is p. That is,

$$P_a = P\left(v \geq k_{a\sigma} / p\right) \quad (3)$$

$$P_r = P\left(v < k_{r\sigma} / p\right) \quad (4)$$

Therefore, the probability of repeating the sampling becomes

$$R = P\left(k_{r\sigma} \leq v \leq k_{a\sigma} / p\right) = 1 - P_a - P_r \quad (5)$$

Then, the probability of accepting a lot under the VRGS plan can be given as (See Balamurali and Jun [2])

$$P_a(p) = \frac{P_a}{1 - P_r + P_a} \quad (6)$$

The probabilities in (3.3) and (3.4) can be expressed as:

$$P_a = P\left\{\overline{X} \leq U - k_{a\sigma} / p\right\} = \Phi(w_a) \quad (7)$$

$$P_r = P\left\{\overline{X} > U - k_{r\sigma} / p\right\} = 1 - \Phi(w_r) \quad (8)$$

where $w_a = \left(\frac{U-\mu}{\sigma} - k_{a\sigma}\right)\sqrt{n_\sigma}$

and $w_r = \left(\frac{U-\mu}{\sigma} - k_{r\sigma}\right)\sqrt{n_\sigma}$

So lot acceptance probability given in (6) is rewritten as $P_a(p) = \dfrac{\Phi(w_a)}{1 - \Phi(w_r) + \Phi(w_a)}$

(9)

$\Phi(w_a)$ and $\Phi(w_r)$ can be found by using normal probability curve.

4. Points of Control and its Relative Slope

Point of control of the OC curve is also termed as Indifference Quality Level (IQL). IQL is defined as “The percentage of variant units in a batch or lot for which, for purposes of acceptance sampling, the probability of acceptance to be restricted to a specific value namely 0.50”. The point (IQL, 0.5) on the OC curve is also called “point of control” (see Hamakar [3]. The fraction nonconforming corresponding to ‘point of control’ or p_0, a dividing point between good and bad lots is a quality level and is obtained such that $P_a(p_0) = 0.50$, where $P_a(p_0)$

is the probability of acceptance at p_0. The relative slope at p_0 on the OC curve, denoted by h_0 is known as the measure of the discriminating power of the OC curve and is obtained as

$$h_0 = \frac{p}{P_a(p)} \frac{dP_a(p)}{dp} \text{ at } p=p_0$$

Where $\frac{dP_a(p)}{dp}$ can be obtained by differentiating $P_a(p)$ with respect to p. Several authors have studied the properties of the sampling plans indexed with IQL (see for example, Charaborty [4] Soundararajan and Devaraj Arumainayagam [5], Balamurali and Kalyanasundaram [6, 7].

5. Selection of VRGS Plan for Given p_0 and its Relative Slope

Table 1 can be used to determine the parameters of the known σ variables RGS plan for specified values of IQL (p_0) and h_0, the relative slope on the OC curve. For example, if p_0 = 4% and h_0=5.649, one can get the parameters of VRGS plan as n=38, $k_{r\sigma} = 1.499$ and $k_{a\sigma} = 2.001$. For the above example, the plan is operated as follows. Take a random sample of size 38 and compute,

$$v = \frac{(U - \overline{X})}{\sigma} \text{ where } \overline{X} = \frac{1}{38}\sum_{i=1}^{38} x_i \qquad (10)$$

Accept the lot if $v > 2.001$ and reject the lot if $v \geq 2.001$. If $1.499 \leq v < 2.001$, then repeat the sampling. In order to show the efficiency of the variables RGS plan, two OC curves are considered. Fig.1. shows the OC curves of the variables single sampling plan and the variables RGS plan both have the same sample size. Variables single sampling plan with ($n_\sigma = 14$ and $k_\sigma = 2.135$) and the variables RGS plan with parameters ($n_\sigma = 14$, $k_{r\sigma} = 1.661$ and $k_{a\sigma} = 2.135$) are considered in this figure.

This figure apparently shows that the variables RGS plan increases the probability of acceptance in the region of principal interest, i.e. for good quality levels and maintains the consumer's risk at poor quality levels. This is an important feature of the variables RGS plan**.**

Table 1: Parameters of the VRGS Plan for Specified p_0 and h_0

p_0 (in %)	h_0	n_σ	$k_{r\sigma}$	$k_{a\sigma}$
0.14	0.57629	3	2.748	3.248
0.30	0.86788	5	2.504	3.003
0.40	0.87028	7	2.548	2.748
0.47	0.96834	7	2.452	2.751
0.54	1.36638	10	2.349	2.749
0.62	1.74133	12	2.252	2.754
0.71	1.94159	12	2.148	2.754
0.94	1.73111	15	2.194	2.498
1.11	2.16426	17	2.100	2.500
1.22	2.57192	18	2.011	2.499
1.39	3.29188	21	1.902	2.503
1.58	3.70647	21	1.804	2.502
1.79	4.31308	22	1.703	2.504
2.02	4.79523	22	1.604	2.504
2.28	4.90141	36	1.748	2.248
2.56	5.11959	32	1.650	2.250
2.87	5.19387	28	1.548	2.249
3.22	5.29492	25	1.452	2.253
3.59	5.48692	23	1.348	2.250
4.00	5.64909	38	1.499	2.001
4.46	6.54438	39	1.401	2.002
4.95	6.97149	35	1.298	2.004
5.48	6.75228	65	1.448	1.749
6.06	7.31162	56	1.348	1.749
6.68	7.50538	47	1.252	1.751
7.35	7.61323	40	1.151	1.752
8.08	8.16228	35	1.049	1.750
8.85	8.45688	75	1.196	1.497
9.68	8.54553	59	1.100	1.500
10.57	9.02388	51	1.004	1.502
11.51	9.29352	44	0.902	1.504
12.51	9.84748	40	0.801	1.499
13.57	10.05427	80	0.948	1.249
14.69	10.48038	65	0.851	1.252
15.87	10.91469	55	0.748	1.253
17.11	11.81228	50	0.652	1.250
18.41	11.46467	41	0.549	1.253
19.77	13.10628	41	0.448	1.249
21.19	13.85744	38	0.353	1.250
21.66	14.09313	63	0.502	1.002

6. Conclusions

In this paper, we have considered the problem of designing the variables repetitive group sampling plan indexed by indifference quality level and its relative slope on the OC curve. It has also been demonstrated that the variables RGS plan has more probability of acceptance at good quality levels and coincide with the OC curve of single sampling variables plan when the quality deteriorates. Table developed in this paper useful for easy selection of parameters for specified p_0 and h_0.

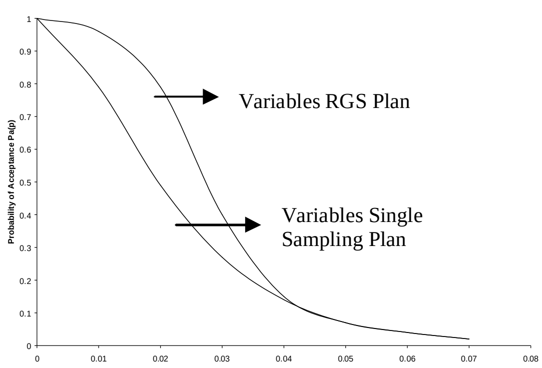

Fig. 1 Comparison OC Curves of Variables Single Sampling Plan and a Variables RGS Plan

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