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INVESTIGATION OF A STRESS CONCENTRATION FACTOR FOR A PLATE WITH DIFFERENT PATTERN OF HOLES-FINITE ELEMENT APPROACH

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ABSTRACT

Three cases-horizontal hole pair in uniaxial loading, hole at the corner and holes at large distance to each other subjected to uniaxial loading are studied in the current work by analytical, experimental and by finite element method(FEM). Results clearly indicate that the finite element approach has yielded fairly encouraging results. This plainly shows FEM can be utilized as a critical apparatus to comprehend the conduct of plates subjected to differed holepatterns subjected to uniaxialloading condition. The current work implies the reliable results given by FEM and also compares the results with the standard experimental data as well as theoretical calculations.

Keywords: *FEM, stress concentration factor, ANSYS, Hole at corner, holes at a far distance*

1. Introduction

Patleet. al. [1] determined stress concentration factors in plate with oblique hole using FEM. Various angle of holes have been considered to evaluate stress concentration factors at such holes. Chaudhuri [2] worked on stress concentration around a part through hole weakening a laminated plate by FEM. Chen [3] developed an element with a circular hole and Piltner [4] introduced one with elliptical and circular holes. These special elements are used at the hole locations at the region near the hole boundary to resolve the stress concentrations while regular elements are used at other locations. The stresses in these elements are determined using a numerical implementation of the afore mentioned complex variable method.

Various methods have been used by different researchers to determine the stresses around the cut out in a plate. Chen [5] used special FEM to obtain the stress concentration around hole in infinite plate. Muskhelishvilli [6] has introduced a complex variable method to solve the problems of theory of elasticity. Savin [8], Lekhnitskii [9], Rao et al. [10], Sharma [10-12], Batista[13], Razaeepazah and Jafari [14-16], Daoust and Hoa [17] etc. have presented the analytical solution to estimate the stress concentration around various shapes of polygonal hole forisotropic and anisotropic plate subjected to remote loading using Muskhelishvilli's [6] complex variable approach to solve the problems of theory of elasticity.

Savin [8], Lekhnitskii [8], Rao et al. [9], Sharma [10–12], Batista [13], Razaeepazah and Jafari [14–16], Daoust and Hoa[17] etc. have presented the analytical solution to estimate the stress concentration around various shapes of polygonal hole for isotropic and anisotropic plate subjected to remote loading using Muskhelishvilli's [6] complex variable approach.

Iwaki [20] worked on stress concentrations in a plate with two unequal circular holes. Ukadgaonker and Rao [21] proposed a general solution for stresses around holes in symmetric laminates by introducing a general form of mapping function and an arbitrary biaxial loading condition in to the boundary conditions. Durelli et al. [24] experimentally evaluated the large strains around elliptical holes and used photo elasticity to determine the stresses at those locations Anubhav and Shital [19] used finite element method to find out stress concentration factor for different loading conditions for vertical and horizontal pair of holes. They found FEM results are closer to the analytical and experimental results.

Based on the literature cited above, no work has been reported on stress concentration for different pattern of holes thus the present study is an attempt to investigate the stress concentration factor for a plate with different pattern of holes using finite element approach.

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Consider a plate with a centrally located hole and the plate is subjected to uniform tensile load at the ends. Stress concentration far from the opening is uniform yet at AA there is a sharp ascent in stress in the region of the gap as appeared in the fig. Stress concentration factor k_t is defined here as,

$$K_t = \frac{\sigma_3}{\sigma_{avg}} \quad (1)$$

Where σ_{avg} at section AA is simply,

$$\frac{P}{t(w-2b)} \quad (2)$$

And

$$\sigma_1 = \frac{P}{wt} \quad (3)$$

Where P= load, t=thickness, w= width of the plate B=radius of the hole, σ= stress

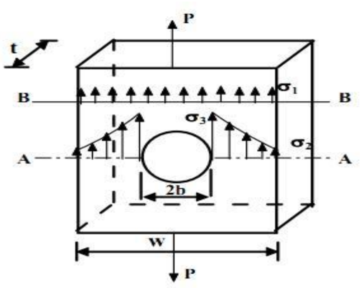

Figure no.1: variation of stress near the discontinuity

This is the theoretical or geometric stress concentration factor and the factor is not affected by the material properties.

In general case it is possible to calculate k_t by using elliptical discontinuity on the plate .In that case k_t is given by

$$K_t = \left[1 + \frac{2b}{a}\right] \quad (4)$$

Where b is the semi-axis of the hole perpendicular to the direction of the hole and a is the semi – axis of the hole in the direction of the hole.

Stress concentration factors may also be obtained using any one of the following experimental techniques:

I. Strain gage method
II. Grid method
III. Brittle coating technique
IV. Photo elasticity method

For more exact estimation numerical techniques like Finite element method might be utilized. Theoretical stress concentration factor for various designs are accessible in handbooks.

In design under fatigue loading, stress concentration factor is used in modifying the values of endurance limit while in design under static loading it simply acts as stress modifier. This means

Actual stress= k_t ×calculated stress.

For ductile materials under static loading effect of stress concentration is not very serious but for brittle materials even for static loading it is important.

It is found that some materials are not very sensitive to the existence of notches or discontinuity. In such cases it is not necessary to use the full value of k_t and instead a reduced value is needed. This is given by a factor known as fatigue strength reduction factor K_f and this is defined as:

K_f = Endurance limit of notch free specimens/Endurance limit of notched specimens

Another term called Notch sensitivity factor, q is often used in design and this is defined as following –

$$q = \frac{(K_f - 1)}{(K_t - 1)} \quad (5)$$

The value of 'q' usually lies between 0 and 1. If q=0, K_f =1 and this indicates no notch sensitivity. If however q=1, then $K_f = K_t$ and this indicates full notch sensitivity.

1.1 The stress concentration factor can be obtained in following ways

a) Analytical method
b) Experimental method
c) Computational method

1.1.1Analytical method

Using the elastic theory concepts in plane stress condition, any applied stress condition can be modeled as plane stress condition and required value of maximum plane stress and minimum plane stress can be obtained.

1.1.2 Experimental method

There are different experimental methods to obtain the maximum strains around a circular hole under different loading conditions.

Some of them are-

Photo-elastic stress analysis techniques

Using standard strain gauge techniques

Using the above techniques different graphs has been plotted. These graphs are used to calculate the stress concentration factor directly and are used as a standard.

1.1.3 Computational method

Current work focuses on maximum stress around the hole by applying the above said loading conditions computationally using ANSYS. The maximum stress obtained is divided by the applied stress called as nominal stress to obtain the stress concentration factor.

2. Experimental Investigation

Table 1. Cases undertaken in the present study

1	Horizontal hole pair in Uniaxial loading
2	Holes at large distance from each other subjected to uniaxial loading
3	Hole at the corner of a plate

2.1Test Specimen Specifications

For Steel Rectangular plate Test Specifications are-

E=210GPA
ν=.3
Thickness=10mm
Length=200mm
Width=100mm
Load=100PA
Hole Radius=10mm
Applied Stress along X axis =100PA
Applied Stress along Y axis =100PA

3. Result and discussion

3.1 Horizontal Hole pair in Uni axial loading

3.1.1 Analytical method

On solving the problem theoretically we have stress concentration factor given as

$$K_t = \left[3 - \left\{0.712 * \frac{d}{l}\right\} + \left\{0.271 * \left(\frac{d}{l}\right)^2\right\}\right] \qquad (6)$$

For d/l = 2/3 **K_t =2.6457**

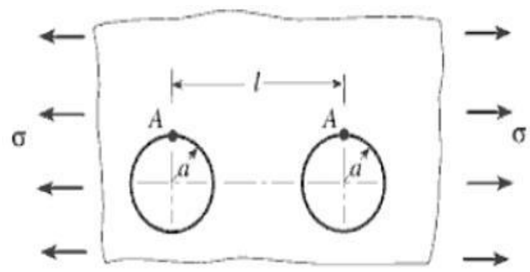

Fig. 2 horizontal hole pair in uniaxial loading

3.1.2 Computational method

On modeling the required specimen on Ansys, analysis was carried and following result was obtained.

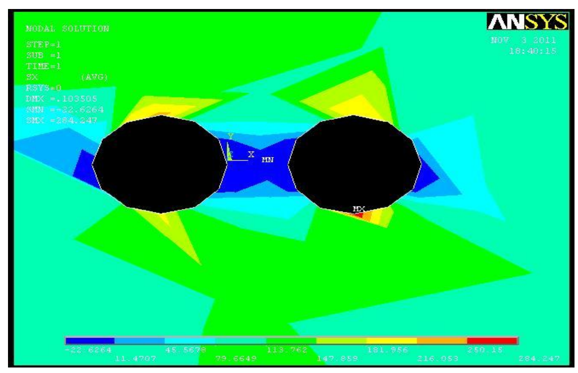

Fig. 3a FEA analysis of plate underuni-axial loading

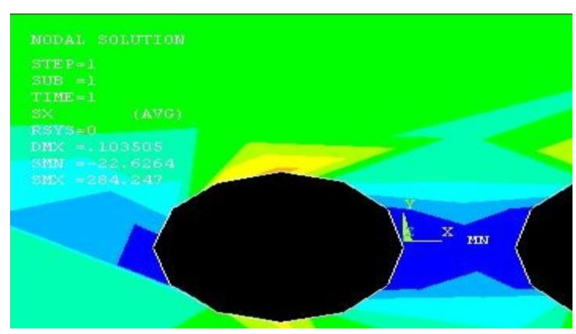

Fig. 3b FEA analysis of plate underuni-axial loading-Kt value

K_{cx}=2.84

The value of Kt obtained by computational method is closer to the value of Kt obtained by analytical method. Computation method will not give exact values because it is based on some assumptions.

3.1.3 Experimental method

Following results in the form of graph are available.

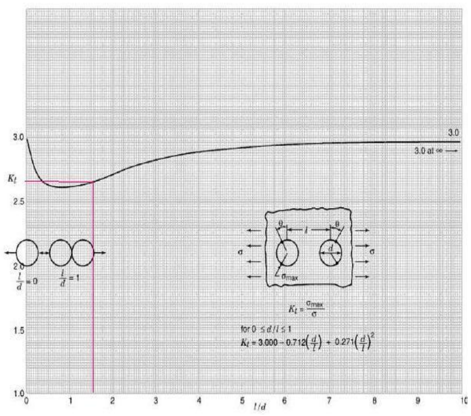

Fig. 4 Standard experimental values as per hand book for uniaxial loading of a plate

For l/d =1.5, $\mathbf{K_t=2.75}$

3.2 Holes at large distance from each other subjected to uniaxial loading

3.2.1 Analytical method

Maximum Stress=3* Applied Stress

K_t =Maximum Stress / Applied Stress

Kt = 3

3.2.2 Computational method

On modeling the required specimen on Ansys, analysis was carried and following result was obtained.

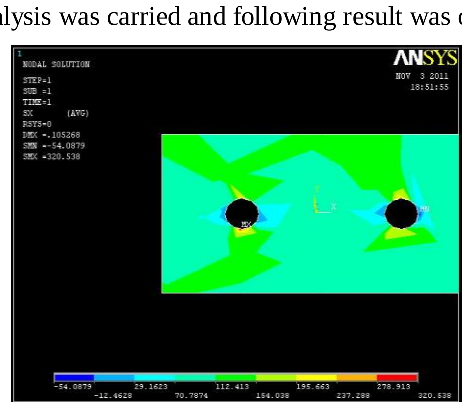

Fig. 5 FEA analysis of plate undergoing uni axial loading

$\mathbf{K_{tc}=3.20}$

3.2.3 Experimentalmethod

Following results in the form of graph are available.

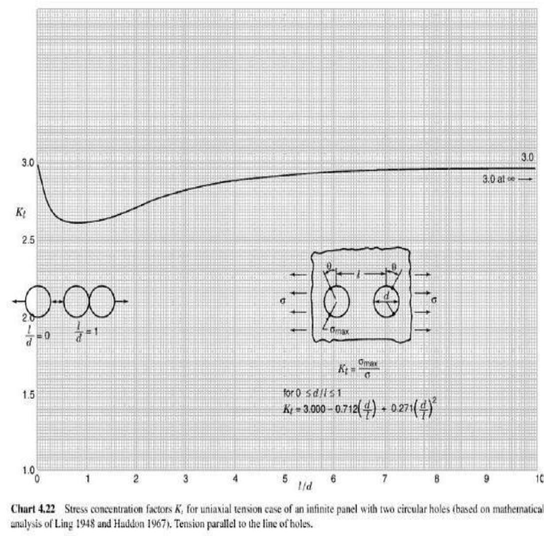

Fig. 6 Standard experimental values as per hand book for uniaxial loading of a plate

K_{te}=3

3.3 Hole at the corner of a plate

3.3.1 Analytical method

Maximum Stress=3* Applied Stress

K_t =Maximum Stress / Applied Stress

K_t =3

3.3.2 Computational method

On modeling the required specimen on Ansys, analysis was carried and following result was obtained

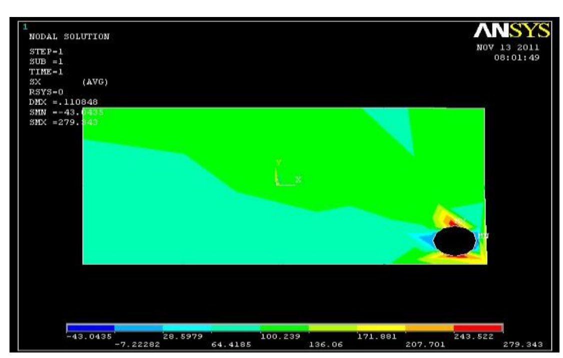

Fig. 7 FEA analysis of plate undergoing uni axial loading

K_{tc}=2.79.

3.3.3 Experimental method

Following results in the form of graph are available.

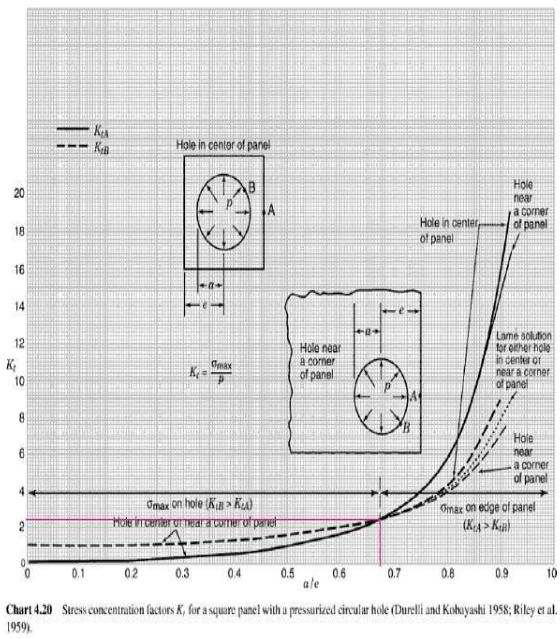

Fig. 8 Standard experimental values as per hand book for uniaxial loading of a plate

K_{te}=2.60

4. Comparison of stress concentration factor for different methods

Table 2. values obtained from experimental, analytical and FEA approach

Sl. No.	Result for	Case 1	Case 2	Case 3
1	Analytical method [AM]	2.6457	3	3
2	Computational method (FEA) [CM]	3.20	3.20	2.79
3	Experimental method [EM]	2.75	3	2.60

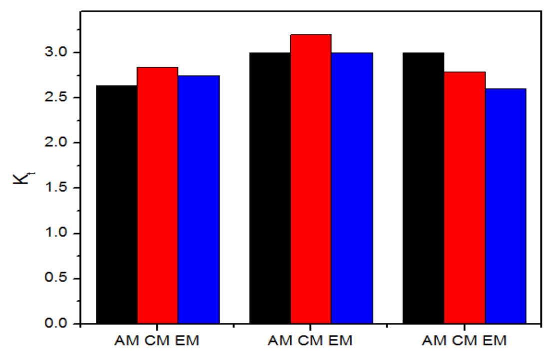

Fig. 9 comparison of all methods for K_t values for 3 cases

Fig 9 shows the comparison of Kt values for different method of analysis for 3 cases respectively.it is observed that values got from FEA method are very close to the analytical and in conversions with experimental method.

5. Analysis of Results

Table 3. Consolidated table of results

Sl. No.	Case	Computational Error	Reference Error
1	Horizontal hole pair in Uniaxial loading	-6.8415	-3.1690
2	Holes at large distance from each other subjected to uniaxial loading	-6.2500	-6.25
3	Hole at the corner of a plate	7.5269	-6.8100
		AVG= -1.85%	AVG= -5.40%

6. Conclusion

- FEM helps to simulate Experimental Procedures and works as an Experimental black box to provide Speedy Results with an average % Error of -5.40 % .
- Average computational time is significantly reduced with the aid of analysis done by the finite element approach.
- Results clearly indicate the acceptance of FEA for evaluation for all three cases.
- FEA can be conveniently used to replace experimental and analytical analysis as the results are very close.[19]
- Stress concentration factor for analytical, experimental and FEA are analysed suitably.

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