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INVESTIGATION OF DEFLECTION AND SHEAR FORCES OF SIMPLY SUPPORTED BEAM USING THEORETICAL NUMERICAL AND SOFTWARE APPROACH

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ABSTRACT

A beam is a key structural member which is widely used in building science as well as different mechanical sector like crane used in industry, loading and unloading of heavy job etc. as such the analysis of beam under loading is important. This paper approaches to if the beam has supported at its two ends and load may be applied anywhere on the beam, the resulting reactions force, deflection, bending moment, shear force can be mathematically estimated using the theory of strength of materials and finite element method(FEM) stiffness matrix is used for validating the software results to the analytical results. Also, aim of this paper is to compare the analytical and mathematical solution with software results. The results may slightly vary with theoretical value and it can be improved by improving quality of mesh.

Key words: Simply Supported Beam, FEM, ANSYS.

1. Introduction

The analysis of static load on an elastic structure has been a topic of interested for over a century. Interested in this problem originated in mechanical as well as civil engineering for the design of crane for industrial purpose, heavyweight lifter rail, railroad high way, and bridge etc.

The classical Bernoulli – Euler theory for deflection of beam connecting the bending moment in the beam with curvature. Rayleigh improved the classical theory with adding a rotary moment of inertia of the cross-section of the beam. Timoshenko extended the theory to include the efficiency of shear deformation. The concluded equation is known as Timoshenko beam equation[2]. Various method of solution has been applied to this problem. Anderson and Dolph gave a general solution and complete analysis of simply supported uniform beam [5,6]. Huang gave frequency equation and normal mode of vibration for the various case of the uniform beam using homogeneous boundary condition. Ritz and Galerkin method was used by Thomas to an obtained frequency of vibration of uniform, tapered and pre-twisted Timoshenko beams with the fixed free end condition. An immense of papers on finite element models have been presented to the analysis of beam by various of the investigator. Many of authors satisfied with their result and few authors claimed that their finite element modal designed to incorporated all boundary condition some of them not able to apply the boundary condition.

The aim of this paper is to simplify the problem by using software which is easy to calculate, as compared to manually or analytical solution, for simplifying this type problem we consider a sample problem and solve that problem by using theoretical formulae and compare the calculated result with the software approach.

2. Finite element method

2.1 Beam elements

Beam elements are obtained by subdivision beam members longitudinally. The simplest Bernoulli-Euler plane beam element, depicted in the figure has two end nodes, 1 and 2 and four degrees of freedom

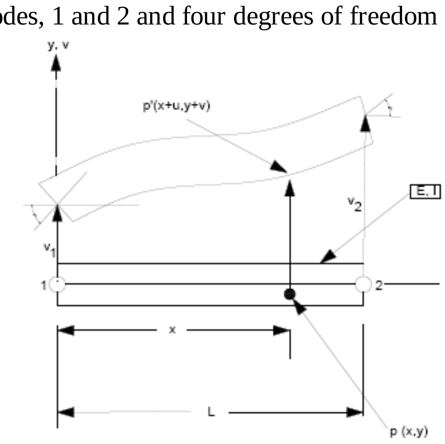

Fig. 1 Beam element

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U=[u1,u2,u3,u4]T

Typically, the degree of freedom of node I are u2-1 and u2i. The degree of freedom u2-1 is the transverse degree of freedom are]u1,u3]T=[v1,v2]T. the rotational degree of freedom are [u2,u4]T=[θ1,θ1]T=[v'1,v'2]T

2.2 Finite element trial functions

The degree of freedom depicted in eq. 2.1 must be used to define uniquely the variation of the transverse displacement v(x) over the element. The c1 continuity requirement stated at the end of the previous section says that both w and the slop θ=v' must be continuous over the entire beam member, and in particular between beam elements C^1 continuity can be trivially satisfied within each element by choosing polynomial interpolation function as shown hereinafter. Then matching the nodal displacement and rotation with adjacent beam elements enforces the necessary inter element continuity.

2.3 Shape functions

The interpolation formula for the beam element may be written as

$U(x,y)= [N_1,N_2,N_3,N_4][u_1,u_2,u_3,u_4]^T = Nu$

The shape function are conveniently written in term of the dimensionless natural co-ordinate system

$$\xi = \frac{2x}{l} - 1$$

which varies from -1 at node 1 (x=0) to +1 at node 2 (x=l); l is the element length

$N_1(\xi)= \frac{1}{4}(1-\xi)^2(2+\xi)$

$N_2(\xi)= \frac{1}{8}L(1-\xi)^2(1+\xi)$

$N_3(\xi)= ¼ (1+\xi)^2(2-\xi)$

$N_4(\xi)=)= \frac{1}{8}L(1+\xi)^2(1-\xi)$

These functions are shown in Fig. 2 and 3.

The curvature k that appears in U can be expressed in term of the nodal displacements by differentiating twice with respect to x.

$$k = \frac{d^2v(x)}{dx^2} = \frac{4}{l^2}\frac{d^2v(\xi)}{d\,\xi^2} = N''u = \frac{4}{l^2}\frac{dN}{d\,\xi}u = Bu$$

Here B=N" is the 1x4 curvature –displacement matrix

$$B = \frac{1}{l}[6\frac{\xi}{l}\,3\,\xi - 1 - 6\frac{\xi}{l}\,3\,\xi + 1]$$

2.4 The stiffness matrix of a prismatic beam

If the bending rigidity EI is constant over element it can be moved out of ξ integral in eq.

$$k_e = \frac{1}{2}\,EIl\int_{-1}^{1} B^T\,Bd\,\xi = \frac{EI}{2L}\int_{-1}^{1}\begin{bmatrix}\frac{6\,\xi}{l}\\ 3\xi-1\\ -\frac{6\,\xi}{l}\\ 3\xi+1\end{bmatrix}\left[6\frac{\xi}{l}3\xi - 1 - 6\frac{\xi}{l}3\xi+1\right]d\xi$$

Expending and integrating over the element yields

$$k_e = \frac{EI}{2l^3}\int_{-1}^{1}\begin{bmatrix}36\xi^2 & 6\xi(3\xi-1)l & -36\,\xi^2 & 6\xi(3\xi+1)l\\ 6\xi(3\xi-1)l & (3\xi-1)^2l^2 & -6\xi(3\xi-1)l & (9\xi^2-1)l^2\\ -36\xi^2 & -6\xi(3\xi-1)l & 36\xi^2 & -6\xi(3\xi+1)l\\ 6\xi(3\xi+1)l & (9\xi^2-1)l^2 & -6\xi(3\xi+1)l & (3\xi-1)^2l^2\end{bmatrix}d\xi$$

$$k_e = \frac{EI}{l^3}\begin{bmatrix}12 & 6l & -12 & 6l\\ 6l & 4l^2 & -6l & 2l^2\\ -12 & -6l & 12 & -6l\\ 6l & 2l^2 & -6l & 4l^2\end{bmatrix}$$

2.5 Shear force and bending moment

Using the bending moment and shear force equations

$M = EI\frac{d^2v}{dx^2}, S = \frac{dM}{dx}$ and $v = Nu$

We get the bending moment and shear force.

$$M = \frac{EI}{l^2}[6\,\xi v_1 + (3\,\xi - 1)l\theta_1 - 8\,\xi v_2 + (3\,\xi + 1)l\theta_2]$$

$$S = \frac{6EI}{l^3}(2v_1 + L\theta_1 - 2v_2 + L\theta_2)$$

The bending moment and shear force value are for the loading as medalled using equivalent point loads. Denoting element and equilibrium load as R_1, R_2, R_3, R_4 we obtain that

$$\begin{bmatrix}R_1\\ R_2\\ R_3\\ R_4\end{bmatrix} = \frac{EI}{l^3}\begin{bmatrix}12 & 6l & -12 & 6l\\ 6l & 4l^2 & -6l & 2l^2\\ -12 & -6l & 12 & -6l\\ 6l & 2l^2 & -6l & 4l^2\end{bmatrix}\begin{Bmatrix}u_1\\ u_2\\ u_3\\ u_4\end{Bmatrix} + \begin{Bmatrix}\frac{-bl}{2}\\ \frac{-bl^2}{12}\\ \frac{-bl}{2}\\ \frac{bl^2}{12}\end{Bmatrix}$$

The shear forces at the two end of the element are S_1=R_1 and S_2=-R_3. The end bending moment are M_1=-R_2 and M_{12}= R_4.

3. Modeling concept

For validating software approach to the theoretical calculation have to modal a CAD or mathematically modal of the prismatic beam whose mass moment of inertia is 2500 cm^2. In the sample, problem beam is made up of steel whose Young's modulus or elastic constant is 200GPa approx.

Load on the prismatic beam applied on the center of the beam the total length is 1000cm and one end is rigidly fixed support and another end can slide on its horizontal axis, applied load magnitude is 20 kN

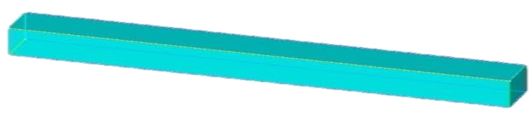

Fig. 2 CAD geometry

For converting the mathematical model to finite element modal, modal have to discretized or mesh FEM modal have been shown in fig below

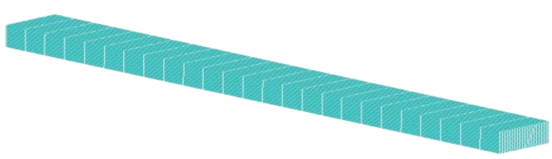

Fig. 3 Finite element modal

4. Validation

Free body diagram shown in figure bellow

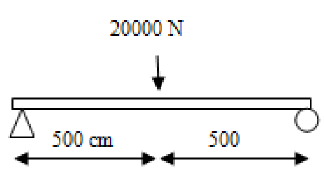

Fig.4 Simply supported beam for analysis.

Nodal displacement vector d= $[u_1, u_2, u_3, u_4, u_5, u_6]^T$
Using equation the element stiffness matrices for element 1 and 2 can be written as

$$k_1 = \frac{20\times10^6\times2500}{500^3}\begin{bmatrix} 12 & 3000 & -12 & 3000 \\ 3000 & 1000000 & -3000 & 5000 \\ -12 & -3000 & 12 & -3000 \\ 3000 & 50000 & -3000 & 1000000 \end{bmatrix}$$

$$k_2 = \frac{20\times10^6\times2500}{500^3}\begin{bmatrix} 12 & 3000 & -12 & 3000 \\ 3000 & 10^6 & -3000 & 5000 \\ -12 & -3000 & 12 & -3000 \\ 3000 & 50000 & -3000 & 10^6 \end{bmatrix}$$

The global stiffness matrix K is now assembled from the element stiffness matrices

$K=k_1+k_2$

$$K = 400\begin{bmatrix} 12 & 3000 & -12 & 3000 & 0 & 0 \\ 3000 & 10^6 & -3000 & 5\times10^5 & 0 & 0 \\ -12 & -3000 & 24 & 0 & -12 & 3000 \\ 3000 & 5\times10^6 & 0 & 2\times10^6 & -3000 & 5\times10^5 \\ 0 & 0 & 12 & -3000 & 12 & -3000 \\ 0 & 0 & 3000 & 5\times10^5 & -3000 & 10^6 \end{bmatrix}$$

The global stiffness matrixes K given above needs to be modified to account for the boundary conditions are: $u_1=u_2=u_5=0$

The element approach is applied to handle the boundary conditions. The rows and column corresponding to degree of freedom 1, 2, and 5 which corresponding to supporting condition are deleted from K matrix. The reduced finite element equations are given as:

$$K = 400\begin{bmatrix} 24 & 0 & 3000 \\ 0 & 2\times10^6 & 5\times10^5 \\ 3000 & 5\times10^5 & 10^6 \end{bmatrix}\begin{Bmatrix} u_3 \\ u_4 \\ u_6 \end{Bmatrix} = \begin{Bmatrix} -2000 \\ 0 \\ 0 \end{Bmatrix}$$

Solution of these equation yield displacements

u_3=-3.646cm, u_4=-0.003125, u_6=0.01250cm

The shear forces and bending moment at each end of an individual member are obtained by using equation The calculation for member one is:

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{Bmatrix} = 4000\begin{bmatrix} 12 & 3000 & -12 & 3000 \\ 3000 & 10^6 & -3000 & 5\times10^5 \\ -12 & -3000 & 12 & -3000 \\ 3000 & 5\times10^5 & -3000 & 10^6 \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ -3.646 \\ -0.003125 \end{Bmatrix}$$

$$= \begin{Bmatrix} 13750N \\ 3750200N.cm \\ -13750N \\ 3125200N.cm \end{Bmatrix}$$

The calculation for member two are:

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{Bmatrix} = 400\begin{bmatrix} 12 & 3000 & -12 & 3000 \\ 3000 & 10^6 & -3000 & 5\times10^5 \\ -12 & -3000 & 12 & -3000 \\ 3000 & 5\times10^5 & -3000 & 10^6 \end{bmatrix}\begin{Bmatrix} -3.646 \\ -0.003125 \\ 0 \\ 0.01250 \end{Bmatrix}$$

$$= \begin{Bmatrix} -6250N \\ -3125200N.cm \\ 6250N \\ 0N \end{Bmatrix}$$

The accuracy of the calculation can be checked by performing an equilibrium analysis of joint two. The resulting moment and resulting applied force is zero.

5. Discussion

5.1 Deformation

Deformation of a beam is 3.646 cm is calculated by the theoretical result and ANSYS calculation is 0.032558 m which are slight deviates which can be solved by increasing the size of mesh the contour diagram of ANSYS for deflection is figured out below.

5.2 Bending/shear force

At fixed point bending moment is high and then it came to zero and then negative the value of bending is the 37495Nm percentage of error is 1% approx bending contour is figured below.

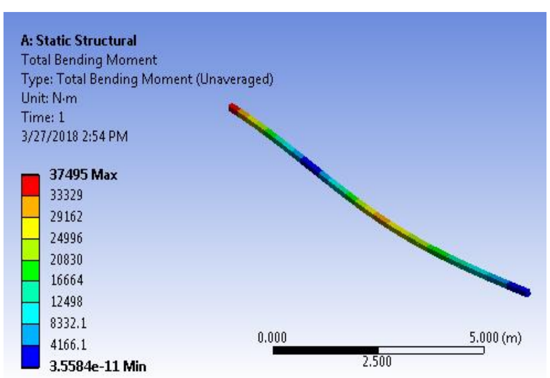

Fig. 4 bending moment contour diagram

Similarly, shear force contour is figured below which is indicated that only two variables are shown by two color maximum shear force is 13749N and minimum 1650N which are very near to actual theoretical results.

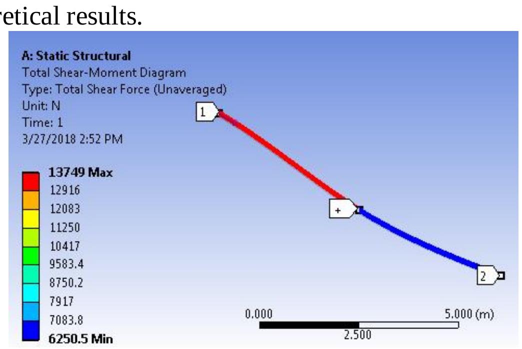

Fig. 5 Shear force contour diagram.

6. Conclusions

The deflection and shear forces are calculated by finite element method as well as software and compare both of results.

The maximum deformation can be achieved 3.255cm on the middle portion of the beam which is approx 1% deviate from the analytical result 3.646cm. in case of shear force up to half of the beam is 13749N and next half is 1650N which is equal to the manual calculation. the maximum bending moment is slightly more deviate to the manual calculation, software result is 3749500Ncm and the manual result is 3750200Ncm

Since results shows, the approach of software to the analytical method results are very close so we can easily calculate for the various section and deferent type of beam for deformation bending and shear force.

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Nomenclature

Abbreviation	Nomenclature
N	Node
U	Displacement/deformation
ξ	Shape function
M	Bending moment
S	Shear force
K	Stiffness matrix
R	Reaction force

